e-ISSN: 2582-5208
International Research Journal of Modernization in Engineering Technology and Science
(Peer-Reviewed, Open Access, Fully Refereed International Journal)
Volume:04/Issue:04/April-2022 Impact Factor- 6.752 www.irjmets.com

RELATIVISTIC TRANSFORMATION OF TEMPERATURE

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ABSTRACT

Relativistic temperature has derived theoretically using the Boltzmann's law assuming an ideal gas. Due to ionization of ideal gases at high temperature the relativistic variation is also proved using the Saha's equation which is applicable at high temperatures. The fine structure correction is also used for correcting the ionization potential term in Saha's equation. Like mass, temperature is also relative if the body attains the velocity comparable to the velocity of light. It is being observed that the following postulates may be formed -

- Thermodynamic laws are valid in all Inertial frames.
- "Absolute Zero" Temperature is constant for all Inertial frames.

I. INTRODUCTION

Emission of total heating power from a surface to be proportional to one fourth of the absolute temperature is called Boltzmann's law. Distribution or measure is probability and this probability system states as a function of energy and the system of temperature. Transformation of temperature is not absolute measurable and relativistic temperature transformation can be measured. The velocity of the moving body is v and the coordinate system decides that the own system is cold or hot. The theory of relativity was discovered in 1905 and the problem of this was solved by Maven Laue, and K.von Mosengeil and they provided the below result

$$T = T_0 \sqrt{(1 - v^2/c^2)} \qquad (1)$$

Where, T_0, the temperature of Kelvin of the system and T is the detecting temperature of the system which is moving. The satisfied formula was taken as the standard formula by solving the doubts by H.Ott where the formula was in inverse form.

$$T = T_0 / \sqrt{(1 - v^2/c^2)} \qquad (2)$$

The satisfying definition of temperature refers to the lack of temperature in the system. The quantity is not measurable of temperature; the only thing that is measurable is the transformation of temperature. This transformation of temperature is measured by a thermometer which is a macroscopic device. Thermometer does not measure the structure of the body and its actual heart. The structure of a system influences the temperature of the body system. Hotness level is related to the observation of thermoscopic states. The existence of thermo scope states is proved by the properties of the thermometer. Thermoplastics states reveal the levels that which are the thermoscopic variable such as length of the mercury thread, resistance and voltage. As per Lorentz transformation of temperature is defined in the form of mathematics term and the formula of measuring transformation temperature is related to reality. The properties of hotness are followed by the general consideration. The invariance of the Kelvin scale is more interesting to reach to the consequence level of transformation of temperature. The formula of the ideal gas is related and dependent on the transformation of temperature. As per the properties of gas pressure is impactful in the transformation of temperature. The pressure of the ideal gas is defined as per Lorentez invariant.

$$P=P_0$$

$$T=T_0$$

Where, 0 reveals the measurement of quantities in a particular point and after taking the transformation of Lorentz, the following equation is to be in the form of r

$$pV = p_0 V_0(1 - v^2/c^2) = RT = R_0 T_0 (1 - v^2/c^2)$$

II. LITERATURE REVIEW

The relation between special relativity and thermodynamics was covered by the physicist Max Plan and Albert Einstein in 1907. The first two laws of thermodynamics are covering the true commoving reference frame of the thermodynamic system. The correct transformation formula for temperature was obtained by Plank. The primary purpose of establishment of this formula was relative oriented. This formula was parallel with the origin of coincidence. Imposing the covariance of these laws under the relativity of the inertial reference frame and both of them are looking for the equations of transformation of temperature. The analysis is to be discussed regarding the thermodynamics substratum. It is being analysed heating reversible and for this transformation of heat the linear momentum of a system for balancing required the below formula:

$$\begin{aligned}\vec{F} &= \frac{d\vec{p}}{dt} = \frac{d}{dt}\left(\vec{p}_{\text{Mechanical}} + \vec{p}_{\text{Heat}}\right) \\ &= \frac{d}{dt}\left(\vec{p}_{\text{M}} + \vec{p}_{\text{H}}\right)\end{aligned}$$

Here, F indicates the force of the extended system and P describes the linear momentum of the system and both of F and P are composed due to mechanical and heating flow. H and M are the subscripts evident. The mistake that Plank has made is indeed subtle. The fact is about the virtue of considering the amount of heat which is transferred. Understanding reason was carried by Plank and the formula given by Plank was not considered by Einstein. The system that is used by Plank and Einstein is not real as this kind of system has no realistic example in the real world. In Spite of having no realistic system in this system the thermodynamic process follows the same theory probably. The velocity as per the equation of virtue

$$\frac{dE}{T} = \frac{dE'}{T'}$$

At present it is considered the transformation of energy in relativistic and transformation of momentum from one temperature to another temperature, element of cylindrical rod at the time of no associating heat.

Observation of different interior performing systems of relativity is to be studied at the same measuring point. Frequent types of this kind of measurement are called distant measurement and the aim of this measurement is determination of the physical quantity that belongs to a moving system by the meaning of measurement that is made at the rest system. Length is the operational method of measurement of distance. On the other hand time and intensity are generally known. Some particular difficulties are encountered by as for the special nature in the field of temperature. Intuitively is not the main problem of quite obvious, the principle is to establish the thermal between the two moving systems. Namely the two moving systems are A and B. The two systems are moving from one to another on the correction test. The common boundary of this test is diathermic or not, which is decided and makes any kind of judgment on the thermal equilibrium. Without changing the boundary the state which is charged a system can create pressure .Without affecting the distribution of charge the system B is surrounded by the metallic envelope and it is done regarding the fact whether it is diathermic or adiabatic. Temperature is measured by the thermometer with respect to the body and in this operation relative observables cannot be performed by the principle of the moving observer. It is the result of measurement of locality and temperature of an object which is moving and its equation. Hence, the temperature measurement cannot be perfect due to this moving condition of the object. They formulas by which the measurement of distance temperature may be calculated are in below:

1. The measured body is to be brought in contact with diathermic in their common rest frame with thermometer.

2. Reconstruct the moving inertial object in reality by reading and applying the transformation rules of the relevant system for the thermoscopic variable using.

III. METHODOLOGY

The observation of 0 is contained that it is contained with ideal gas and the bottoms of this ideal gas are free in moving and the energy of this state is accurate. The energy after the collation of two atoms is described by the Bolzmann's distribution theory

$$n' = N \exp(-E/kT)$$

Here, N reveals the total number of atoms of the system.

T is used for the temperature of the regarding system

K is used as the constant of Boltzmann

And He describes the state energy level.

Two energy systems are considered due to the high energy system and the same density number is used for the two observations. Some problems are connected with the context of relativistic temperature. First parameter is to be set up in a scalar or a vector is to be cleared. After the observation O, difference of energy level is to be described as per the structure of atoms and it is defined between the two consecutive potential differences.

$$X_r = -mc^2Z^2\alpha^2\,(1/\,2(r+1)^2 - 1/\,2r^2)$$

Similarly the observation of realistic of the difference of energy level is corrected as

$$E_{r,j} = -mc^2[1-(Z^2\alpha^2)/(2r^2)-(Z^4\alpha^4)/(2r^4)\{r/(j+1/2)-3/4\}]$$

The same observations are considered to be as an experiment and the particular note is to be noted for the discussion. The theory of Boltzmann is to be described after doing the two experimental observations. The theory is to be considered on the basis of ground energy levels. The comparison of two ground energy levels is to be used as the potential difference and by using this potential difference the equation is to be made simple. In both cases the comparison will make then below equation

$$T = T'/\beta$$

IV. DISCUSSION

Consider two observers. The 1st Observer (O) is at rest (which will be considered as a "stationary observer") and the other Observer (O') (which is moving at a velocity "v" with respect to O).Both frame of references are inertial.

Observer O is observing in a container containing an ideal gas whose atoms are free to move and acquire any possible energy state. The collision between the atoms and the walls of the container is perfectly elastic in nature.

According to Boltzmann Law of distribution of atoms in different energy state

$$n' = N\,exp(-E/kT) \qquad \dots (a)$$

Where n = Number of atoms in energy state E

N = Total number of atoms in the system

T = Temperature of the closed container

k = Boltzmann's constant

exp = exponential

E = Energy state

For another observer O' according to the Boltzmann law

$$n' = N\,exp(-E/kT)$$

Total energy of a particle

E = K.E. + P.E.

But P.E. =0, as particle is free to move in the container.

And hence

$$n' = N\ exp(-K.E'/kT') \qquad \dots (b)$$

Similarly for observer O

$$n = N\ exp(-K.E/kT) \qquad \dots (c)$$

Assume E_{max} to be unity for O', the maximum probable energy that can be achieved by the atom is 1.

i.e. $K.E'_{max} = 1$ ... (d)

Now the number of atoms present in the state at half the energy for the observer O' is

$$n'_{1/2} = N\,exp(-K.E'/2kT') \qquad \dots (e)$$

And for the observer O

$$n_{1/2} = N\,exp(-K.E/2kT) \qquad \dots (f)$$

The maximum energy level for the observer O will not be 1 but 1 times β.

i.e. $E_{max} = \beta * 1$

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Where $\beta = (1 - v^2/c^2)^{1/2}$ [Lorentz Factor]… (g)

So the $K.E._{max}$ for observer O is

$K.E._{max} = \beta * 1$

$K.E._{max} = \beta * K.E.'_{max}$ … (h)

Since the number of atoms in the half of max state in O' is equal to the number of atom in the half of max Energy State O

$$n'_{1/2} = n_{1/2} \quad \text{… (i)}$$

Solving equations (5), (6), (8) & (9) , we have

$$exp(-K.E'/2kT') = exp(-K.E/2kT)$$

And hence

$$T = T'/\beta \quad \text{… (j)}$$

High Energy Thermodynamic Systems

For high energy system (that is high temperature system), we consider two observer O and O' is observing the same system a system. Let first observer (O) is at rest (which will be considered as a "stationary system") and the other observer (O') (which is moving at a velocity "v" with respect to O). The same procedure is used and the number densities are compared for the two observers.

Now from the Saha's Equation[1]

$$(n_{(r+1)}/n_r)n_e = (G_{(r+1)}/G_r)\ g_e\ [(2\pi\, m_e\, kT)^{3/2}/h^3]\ exp(X_r/kT) \quad \text{… (k)}$$

Where,

n_{r+1}: : Density of atoms in ionization state r+1 (m−3)

n_r: : Density of atoms in ionization state r

n_e: : Density of electrons

G_{r+1} : Partition function[2] of ionization state r+1.

$$G_r = \sum_{j=0}^{r} g_j\ exp(-E_j/kT) \quad \text{… (l)}$$

G_r : Partition function of ionization state r:

g_e = 2: : Statistical weight of the electron

m_e : Mass of the electron

X_r : Ionization potential of state r (to reach state r+1), ground level to ground level

T : Temperature

h :Planck's constant

For observer O, potential difference between the two consecutive energy levels can be defined as the difference of energy level described by fine structure[3] of atoms.

i.e.

$$X_r = -mc^2Z^2\alpha^2\ (1/\ 2(r+1)^2 - 1/\ 2r^2) \quad \text{… (m)}$$

Where

Z is the atomic number of atom

M is the mass of electron

r is the energy level of the atom

and

α is the fine structure constant[4]

= e^2/hc (c is the speed of light in vacuum)

And then by by using Saha's equations (j) and (m), we have

$$(n_{(r+1)}/n_r)n_e = (G_{(r+1)}/G_r)\ g_e\ [(2\pi\, m_e\, kT)^{3/2}/h^3]\ exp[(-mc^2Z^2\alpha^2)\ (1/\ 2(r+1)^2 - 1/\ 2r^2)/kT] \quad \text{…(n)}$$

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Similarly for the relativistic observer O' the energy difference in states can be defined by fine structure splitting correction [5].

$$E_{r,j} = -mc^2[1 - (Z^2\alpha^2)/(2r^2) - (Z^4\alpha^4)/(2r^4)\{r/(j+1/2) - 3/4\}] \quad \text{... (o)}$$

here j is the azimuthal quantum number

Now the ionization potential for the relativistic observer O' will be [6]

$$X'_r = E'_{r+1} - E'_r$$

$$X'_r = -m'c^2[1 - (Z^2\alpha^2)/(2(r+1)^2) - (Z^4\alpha^4)/(2(r+1)^4)\ (((r+1)/(j+1/2)) - 3/4)] - [-m'c^2\{1 - (Z^2\alpha^2)/(2r^2) - (Z^4\alpha^4)/(2r^4)\ (r/(j+1/2) - 3/4)\}] \quad \text{... (p)}$$

Where

$$m' = m * \beta$$

Since the comparison of ground levels of energy level are used as potential difference, we would take j=0*.

On simplifying the above equations we get

$$X'_r = -m'c^2\,[(Z^2\alpha^2)\,(1/2(r+1)^2 - 1/2r^2) - (Z^4\alpha^4)\,(1/2(r+1)^4 - 1/2r^4)\ (2n - 3/4) + (Z^4\alpha^4)/(r+1)^4\,]$$

But since the order of the term

$$\left[-m'c^2\left\{Z^4\alpha^4\left(1/2(r+1)^4 - 1/2(r^4)\right)(2n-3/4) + (Z^4\alpha^4)/(r+1)^4\right\}\right]$$

Is less than 10^{-2} the exponential of the term can be neglected as it will result approaches to 1 i.e.

$$exp\left[-m'c^2\left\{Z^4\alpha^4\left(1/2(r+1)^4 - 1/2(r^4)\right)(2n-3/4) + (Z^4\alpha^4)/(r+1)^4\right\}\right] \approx 1 \quad \text{... (q)}$$

Therefore from equation (11)

Saha's equation for the relativistic observer O' will be

$$(n'_{r+1}\,n'_e)/(n'_r) = \left[(G'_{r+1}\,g'_e)/(G'_r)\right]\left[(2\pi\, m'_e kT')^{(3/2)}/h^3\right] \exp[(-m'c^2Z^2\alpha^2\,(1/(r+1)^2 - 1/r^2))/(2kT')] \quad \text{... (r)}$$

And the proof of Saha's equation is based upon the consideration of ground state of different energy levels

$$n'_r = n_r \quad \text{and}$$

$$n'_e = n_e$$

Since the same system is observed by both the observer O and O', n_{r+1}, n_r, n_e is same for both (they are the numbers of particles and it is to be noted that thought experiment it is similar as described above in proof using Boltzmann constant) i.e.

$$n'_{r+1} = n_{r+1}$$

But as the value of (kT) is very small (in evs) and when used for calculation of partition function for both the observer O and O', therefore

$$G_{r+1}/G_r \approx g_{r,0}/g_{r+1,0} \approx (G'_{r+1})/(G'_r) \quad \text{... (s)}$$

Now, using equations (m), (r) and (s), we have

$$g_e\,[(2\pi\, m_e kT)^{3/2}/h^3]\ \exp[(-mc^2Z^2\alpha^2\,(1/(r+1)^2 - 1/r^2))/2kT = g'_e\ ((2\pi\, m'_e kT')^{3/2}/h^3)\ \exp[(-m'c^2Z^2\alpha^2(1/(r+1)^2 - 1/r^2))/2kT'] \quad \text{... (t)}$$

putting $g'_e = g_e = 2$ and $m'_e = \beta * m_e$ in above equation and comparing the both sides of equation we get

$$\mathbf{T = T'/\beta}$$

V. CONCLUSION

Using both Boltzmann distribution law and the Saha's equation for higher energy particles we observed that for two inertial observers (one in rest and the other moving with a velocity 'v') will measure the relative temperature with respect to each other. The relative temperature is velocity dependent.

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