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INTERVAL VALUED INTUITIONISTIC FUZZY DERIVATIONS OF KU-IDEAL IN KU-ALGEBRA

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ABSTRACT

This paper presents the concept of interval-valued intuitionistic fuzzy derivations of KU-ideals in KU algebras. We have introduced the notion of interval-valued intuitionistic fuzzy derivation sets in KU algebras and provided examples to illustrate this concept. Additionally, interval-valued intuitionistic fuzzy derivation sets under homomorphisms in KU algebras and also defined the Cartesian product of interval-valued intuitionistic fuzzy derivation of KU-ideals in KU algebras.

Keywords: KU Algebras, Interval Valued Intuitionistic Fuzzy Derivation Set, Interval Valued Intuitionistic Fuzzy Left(Right)-Derivations KU-Ideals, Homomorphisms Of KU-Ideals, Products Of Interval Valued Intuitionistic Fuzzy Derivations KU-Ideals.

I. INTRODUCTION

The notion of a fuzzy set was first introduced by L.A.Zadeh[13] in 1965 . K.T Atanassov[14,15] defined the intuitionistic fuzzy subset which is the generalisation to fuzzy set further the intuitionistic fuzzy set was extended to intuitionistic fuzzy ideal which was defined by Basnet and Benerjee[16,17].Some researchers introduced the generalisation of Fuzzy sets that is called as the interval valued intuitionistic fuzzy sets.The theory of interval valued intuitionistic fuzzy sets was first introduced by Atanassov and also established the operators and its properties.

The two classes of algebras of logic are BCK and BCI-algebras which were introduced by Imai and Iseki[2,3,4] . The concept of derivations, originally applied in ring and near-ring theory, was extended to BCI algebras by Jin and Xin[5]. They also introduced the notion of regular derivations in this context. Building on this work, Hamza and Al Shehri[1] defined left derivations in BCI algebras and explored the properties of regular left derivations. Furthermore, G. Muhiuddin etl[9,10] developed the concept of (α, β)-derivations in BCI algebras. Then a new algebraic structure called the KU – algebra was introduced by C.Prabpayak and U.Leerawat[11,12] .More recently, Mostafa et al[7,8] introduced (l, r) – (r, l)-derivations in KU algebras and investigated their related properties.Samy M.Mostafa and Ahmed Abd-eldayem[6,18,19] presented Fuzzy KU-ideals in KU-algebra , Intuitionistic Fuzzy KU-ideals in KU – algebra and furthermore he introduced the Fuzzy derivations KU-ideals on KU-algebra.

This paper contains some basic definitions then it has interval valued intuitionistic fuzzy derivations KU-ideals of KU- algebra and the concept of image and preimage of interval valued intuitionistic fuzzy derivatives of KU – algebra is discussed. Finally the product of interval valued intuitionistic fuzzy left derivations KU- ideals is presented.

II. PRELIMINARIES

Definition 2.1[19] :

Let X be a KU-algebra and $d : X \to X$ be self map .A non - empty subset A of a KU-algebra X is called left derivations KU ideal of X if it satisfies the following conditions:

(1) $0 \in A$

(2) $d(x) * (y * z) \in A$, $d(y) \in A$ implies $d(x * z) \in A$, for all $x , y , z \in X$.

Definition 2.2[19] :

Let X be a KU-algebra and d : X → X be self map .A non - empty subset A of a KU-algebra X is called right derivations KU ideal of X if it satisfies the following conditions:

(1) 0 ∈ A,

(2) x * d(y * z) ∈ A , d(y) ∈ A implies d(x * z) ∈ A , for all x , y , z ∈ X.

Definition 2.3[19]:

Let X be a KU-algebra and d : X → X be self map .A non - empty subset A of a KU-algebra X is called derivations KU -ideal of X if it satisfies the following conditions:

(1) 0 ∈ A,

(2) d(x * (y * z)) ∈ A , d(y) ∈ A implies d(x * z) ∈ A , for all x , y , z ∈ X

Definition 2.4 [11,12]

A non empty subset A of KU-algebra X is called ideal of X if it is satisfied the following conditions:

(i) 0∈ A

(ii) y * z ∈ A , y ∈ A implies z ∈ A ∀ y,z ∈ X .

Definition 2.5 [6]

Let X be a KU-algebra , a fuzzy set μ in X is called a fuzzy KU-ideal of X if it satisfies the following conditions:

(F1) μ (0) ≥ μ (x) ,

(F2) μ (x * z) ≥ min {μ (x * (y * z)), μ (y)}.

Definition 2.6[7]

Let X be a KU-algebra. A self map d : X → X is a left –right derivation (briefly, (l,r) -derivation) of X if it satisfies the identity

d(x * y) = (d(x) * y) ∧ (x * d(y)) ∀ x, y∈ X

If d satisfies the identity

d(x * y) = (x * d(y)) ∧ (d(x) * y) ∀ x, y∈ X

d is called right-left derivation (briefly, (r,l) -derivation) of X . Moreover, if d is both

(l,r) and (r,l) - derivation then d is called a derivation of X .

Definition 2.7[7]

A derivation of KU-algebra is said to be regular if d(0) = 0 .

Proposition 2.8 [7]

Let X be a KU-algebra with partial order ≤ , and let d be a derivation of X . Then the following hold ∀ x, y ∈ X :

(i) d(x) ≤ x .

(ii) d(x * y) ≤ d(x) * y .

(iii) d(x * y) ≤ x * d(y) .

(v) d(x * d(x)) = 0 .

(vi) $d^{-1}(0)$ = {x ∈ X | d(x) = 0} is a sub algebra of X .

Definition 2.9 [11,12]

Let X be a set with a binary operation * and a constant 0. (X ,*, 0) is called KU-algebra if the following axioms hold : ∀ x,y,z ∈ X :

(KU1) (x * y)* [(y * z)*(x * z)] = 0

(KU2) x*0 = 0

(KU3) 0*x = x

(KU4) if x * y = 0 = y * x implies x = y

Define a binary relation ≤ by : x ≤y iff y * x = 0 , we can prove that (X ,≤)

is a partially ordered set .

Throughout this article, X will denote a KU-algebra unless otherwise mentioned.

III. INTERVAL VALUED INTUITIONISTIC FUZZY DERIVATIONS KU- IDEALS OF KU-ALGEBRAS

In this section, we will discuss and investigate a new notion called interval valued intuitionistic fuzzy- left derivations KU - ideals of KU - algebras and study several basic properties which are related to interval valued intuitionistic fuzzy left derivations KU - ideals.

Definition 3.1 :

Let X be a KU-algebra and d : X → X be self map. An interval valued intuitionistic fuzzy set μ : X → [0,1] in X is called an interval valued intuitionistic fuzzy left derivations KU-ideal(briefly,(F,l)-derivation) of X if it satisfies the following conditions:

(IFL1) $\alpha_A{}^L(0) \geq \alpha_A{}^L(x)$, $\beta_A{}^L(0) \leq \beta_A{}^L(x)$

(IFL2) $\alpha_A{}^U(0) \geq \alpha_A{}^U(x)$, $\beta_A{}^U(0) \leq \beta_A{}^U(x)$

(IFL3) $\alpha_A{}^L(d(x*z)) \geq \min\{\alpha_A{}^L(d(x)*(y*z)), \alpha_A{}^L(d(y))\}$,

$\beta_A{}^L(d(x*z)) \leq \max\{\beta_A{}^L(d(x)*(y*z)), \beta_A{}^L(d(y))\}$

(IFL4) $\alpha_A{}^U(d(x*z)) \geq \min\{\alpha_A{}^U(d(x)*(y*z)), \alpha_A{}^U(d(y))\}$,

$\beta_A{}^U(d(x*z)) \leq \max\{\beta_A{}^U(d(x)*(y*z)), \beta_A{}^U(d(y))\}$

Definition 3.2 :

Let X be a KU-algebra and d : X → X be self map. An interval valued intuitionistic fuzzy set μ : X → [0,1] in X is called an interval valued intuitionistic fuzzy right derivations KU-ideal(briefly, (F,r) -derivation) of X if it satisfies the following conditions:

(IFR1) $\alpha_A{}^L(0) \geq \alpha_A{}^L(x)$, $\beta_A{}^L(0) \leq \beta_A{}^L(x)$,

(IFR2) $\alpha_A{}^U(0) \geq \alpha_A{}^U(x)$, $\beta_A{}^U(0) \leq \beta_A{}^U(x)$,

(IFR3) $\alpha_A{}^L(d(x*z)) \geq \min\{\alpha_A{}^L((x)*d(y*z)), \alpha_A{}^L(d(y))\}$,

$\beta_A{}^L(d(x*z)) \leq \max\{\beta_A{}^L((x)*d(y*z)), \beta_A{}^L(d(y))\}$,

(IFR4) $\alpha_A{}^U(d(x*z)) \geq \min\{\alpha_A{}^U((x)*d(y*z)), \alpha_A{}^U(d(y))\}$,

$\beta_A{}^U(d(x*z)) \leq \max\{\beta_A{}^U((x)*d(y*z)), \beta_A{}^U(d(y))\}$

Definition 3.3 :

Let X be a KU-algebra and d : X → X be self map. An interval valued intuitionistic fuzzy set μ : X → [0,1] in X is called an interval valued intuitionistic fuzzy derivations KU-ideal ,if it satisfies the following conditions

(IFD1) $\alpha_A{}^L(0) \geq \alpha_A{}^L(x)$, $\beta_A{}^L(0) \leq \beta_A{}^L(x)$

(IFD2) $\alpha_A{}^U(0) \geq \alpha_A{}^U(x)$, $\beta_A{}^U(0) \leq \beta_A{}^U(x)$,

(IFD3) $\alpha_A{}^L(d(x*z)) \geq \min\{\alpha_A{}^L(d(x)*(y*z)), \alpha_A{}^L(d(y))\}$,

$\beta_A{}^L(d(x*z)) \leq \max\{\beta_A{}^L(d(x)*(y*z)), \beta_A{}^L(d(y))\}$,.

(IFD4) $\alpha_A{}^U(d(x*z)) \geq \min\{\alpha_A{}^U(d(x)*(y*z)), \alpha_A{}^U(d(y))\}$,

$\beta_A{}^U(d(x*z)) \geq \max\{\beta_A{}^U(d(x)*(y*z)), \beta_A{}^U(d(y))\}$

Example 3.4 :

Let X = { 0,1,2,3.4 } be a set in which the operation * is defined as follows:

we can prove that (X, *, 0) is a KU-algebra.

*	0	1	2	3	4
0	0	1	2	3	4
1	0	0	2	3	3
2	0	1	0	1	4
3	0	0	0	0	3
4	0	0	0	0	0

Define a self map d : X → X by

$$d(x) = \begin{cases} 0 & , \ if\ x = 0,1,2,3 \\ 4 & , \quad if\ x = 4 \end{cases}$$

$\alpha_A{}^L(0) = 0.5$, $\alpha_A{}^L(1) = \alpha_A{}^L(2) = \alpha_A{}^L(3) = \alpha_A{}^L(4) = 0.3$

$\alpha_A{}^L(0) = 0.6$, $\alpha_A{}^L(1) = \alpha_A{}^L(2) = \alpha_A{}^L(3) = \alpha_A{}^L(4) = 0.2$

$\beta_A{}^L(0) = 0.4$, $\beta_A{}^L(1) = \beta_A{}^L(2) = \beta_A{}^L(3) = \beta_A{}^L(4) = 0.6$

$\beta_A{}^L(0) = 0.3$, $\beta_A{}^L(1) = \beta_A{}^L(2) = \beta_A{}^L(3) = \beta_A{}^L(4) = 0.7$

Thus by calculation it satisfies the conditions of interval valued intuitionistic fuzzy left(right) derivations KU-ideals of KU algebra X.

Lemma 3.5:

Let μ be an interval valued intuitionistic fuzzy left derivations KU - ideal of KU - algebra X , if the inequality , $x * y \leq d(z)$ holds in X, then

$\alpha_A{}^L(d(y)) \geq \min\{\alpha_A{}^L(d(x)), \alpha_A{}^L(z)\}$.

$\alpha_A{}^U(d(y)) \geq \min\{\alpha_A{}^U(d(x)), \alpha_A{}^U(z)\}$.

$\beta_A{}^L(d(y)) \leq \max\{\beta_A{}^L(d(x)), \beta_A{}^L(z)\}$.

$\beta_A{}^U(d(y)) \leq \max\{\beta_A{}^U(d(x)), \beta_A{}^U(z)\}$.

Proof.

Assume that the inequality $x*y \leq d(z)$ holds in X , then

$d(z)*(x*y) = 0$, $(z)*(x*y) = 0$, since $d(z) \leq z$ from (Proposition 2.13(i)) and by(FL2),

we have

$\alpha_A{}^L(d(z*y)) \geq \min\{\alpha_A{}^L(d(z)*(x*y)), \alpha_A{}^L(d(x))\}$

$= \min\{\alpha_A{}^L(0), \alpha_A{}^L(d(x))\}$

$= \alpha_A{}^L(d(x))$

$\beta_A{}^L(d(z*y)) \leq \max\{\beta_A{}^L(d(z)*(x*y)), \beta_A{}^L(d(x))\}$

$= \min\{\beta_A{}^L(0), \beta_A{}^L(d(x))\}$

$= \beta_A{}^L(d(x))$

$\alpha_A{}^U(d(z*y)) \geq \min\{\alpha_A{}^U(d(z)*(x*y)), \alpha_A{}^U(d(x))\}$

$= \min\{\alpha_A{}^U(0), \alpha_A{}^U(d(x))\}$

$= \alpha_A{}^U(d(x))$

$\beta_A{}^U(d(z*y)) \leq \max\{\beta_A{}^U(d(z)*(x*y)), \beta_A{}^U(d(x))\}$

$= \max\{\beta_A{}^U(0), \beta_A{}^U(d(x))\}$

$= \beta_A{}^U(d(x))$

Put z=0 , we have

$\alpha_A{}^L(d(0*y)) = \alpha_A{}^L(d(y)) \geq \min\{\alpha_A{}^L(x*y), \alpha_A{}^L(d(x))\}$............ (i),

$\alpha_A{}^U(d(0*y)) = \alpha_A{}^U(d(y)) \geq \min\{\alpha_A{}^U(x*y), \alpha_A{}^U(d(x))\}$............ (ii)

$\beta_A{}^L(d(0*y)) = \beta_A{}^L(d(y)) \leq \max\{\beta_A{}^L(x*y), \beta_A{}^L(d(x))\}$............ (iii)

$\beta_A{}^U(d(0*y)) = \beta_A{}^U(d(y)) \leq \max\{\beta_A{}^U(x*y), \beta_A{}^U(d(x))\}$............ (iv)

but $\alpha_A{}^L(x*y) \geq \min\{\alpha_A{}^L(x*(z*y)), \alpha_A{}^L(z)\}$

$= \min\{\alpha_A{}^L(z*(x*y)), \alpha_A{}^L(z)\}$

$= \min\{\alpha_A{}^L(0), \alpha_A{}^L(z)\} = \alpha_A{}^L(z)$ ....(v)

$\beta_A{}^L(x*y) \leq \max\{\beta_A{}^L(x*(z*y)), \beta_A{}^L(z)\}$

$= \max\{\beta_A{}^L(z*(x*y)), \beta_A{}^L(z)\}$

$= \max\{\beta_A{}^L(0), \beta_A{}^L(z)\} = \beta_A{}^L(z)$ ....(vi)

$\alpha_A{}^U(x*y) \geq \min\{\alpha_A{}^U(x*(z*y)), \alpha_A{}^U(z)\}$

$= \min\{\alpha_A{}^U(z*(x*y)), \alpha_A{}^U(z)\}$

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$=\min\{\alpha_A{}^U(0), \alpha^U(z)\} = \alpha_A{}^U(z)$ ....(vii)

$\beta_A{}^U(x * y) \le \max\{\beta_A{}^U(x * (z * y)), \beta_A{}^U(z)\}$

$= \max\{\beta_A{}^U(z * (x * y)), \beta_A{}^U(z)\}$

$=\max\{\beta_A{}^U(0), \beta^U(z)\} = \beta_A{}^U(z)$ ....(viii)

From (i), (v), we get $\alpha_A{}^L(d(y)) \ge \min\{\alpha_A{}^L(d(x)), \alpha_A{}^L(z)\}$.

From (ii), (vii), we get $\alpha_A{}^U(d(y)) \ge \min\{\alpha_A{}^U(d(x)), \alpha_A{}^U(z)\}$.

From (iii),(vi), we get $\beta_A{}^L(d(y)) \le \max\{\beta_A{}^L(d(x)), \beta_A{}^L(z)\}$.

From (iv),(viii), we get $\beta_A{}^U(d(y)) \le \max\{\beta_A{}^U(d(x)), \beta_A{}^U(z)\}$.

This completes the proof.

Lemma 3.6:

If $((\alpha_A{}^L, \beta_A{}^L),(\alpha_A{}^U, \beta_A{}^U))$ is an interval valued intuitionistic fuzzy left derivations KU - ideal of KU - algebra X and if $x \le d(y)$, then

$\alpha_A{}^L(d(x)) \ge \alpha_A{}^L(d(y))$.

$\alpha_A{}^U(d(x)) \ge \alpha_A{}^U(d(y))$.

$\beta_A{}^L(d(x)) \le \beta_A{}^L(d(y))$.

$\beta_A{}^U(d(x)) \le \beta_A{}^U(d(y))$.

Proof .

If $x \le d(y)$,then $d(y) * x = 0$, $y*x=0$ since $d(y) \le y$ (from Proposition 2.13(i)) this together with $0 * x = x$ and $\alpha_A{}^L(0) \ge \alpha_A{}^L(y)$ and $\alpha_A{}^U(0) \ge \alpha_A{}^U(y)$, we get

$\alpha_A{}^L(d(0 * x)) = \alpha_A{}^L(d(x)) \ge \min\{\alpha_A{}^L(d(0) * (y * x)), \alpha_A{}^L(d(y))\}$

$= \min\{\alpha_A{}^L(0 * 0), \alpha_A{}^L(d(y))\}$

$= \min\{\alpha_A{}^L(0), \alpha_A{}^L(d(y))\}$

$= \alpha_A{}^L(d(y))$.

$\alpha_A{}^U(d(0 * x)) = \alpha_A{}^U(d(x)) \ge \min\{\alpha_A{}^U(d(0) * (y * x)), \alpha_A{}^U(d(y))\}$

$= \min\{\alpha_A{}^U(0 * 0), \alpha_A{}^U(d(y))\}$

$= \min\{\alpha_A{}^U(0), \alpha_A{}^U(d(y))\}$

$= \alpha_A{}^U(d(y))$.

$\beta_A{}^L(0) \le \beta_A{}^L(y)$ and $\beta_A{}^U(0) \le \beta_A{}^U(y)$, we get

$\beta_A{}^L(d(0 * x)) = \beta_A{}^L(d(x)) \le \max\{\beta_A{}^L(d(0) * (y * x)), \beta_A{}^L(d(y))\}$

$= \max\{\beta_A{}^L(0 * 0), \beta_A{}^L(d(y))\}$

$= \max\{\beta_A{}^L(0), \beta_A{}^L(d(y))\}$

$= \beta_A{}^L(d(y))$.

$\beta_A{}^U(d(0 * x)) = \beta_A{}^U(d(x)) \le \max\{\beta_A{}^U(d(0) * (y * x)), \beta_A{}^U(d(y))\}$

$= \max\{\beta_A{}^U(0 * 0), \beta_A{}^U(d(y))\}$

$= \max\{\beta_A{}^U(0), \beta_A{}^U(d(y))\}$

$= \beta_A{}^U(d(y))$.

Proposition 3.7 :

The intersection of any set of interval valued intuitionistic fuzzy left derivations KU - ideals of KU –algebra X is also an interval valued intuitionistic fuzzy left derivations KU - ideal.

Proof.

Let $\{(\alpha_i{}^L, \beta_i{}^L),(\alpha_i{}^U, \beta_i{}^U)\}$ be a family of interval valued intuitionistic fuzzy left derivations KU - ideals of KU- algebra X , then for any $x, y, z \in X$,

$(\cap\alpha_i{}^L)(0) = \inf(\alpha_i{}^L(0)) \ge \inf(\alpha_i{}^L(d(x)))\ (\cap\alpha_i{}^L)(d(x))$

$(\cap\alpha_i{}^U)(0) = \inf(\alpha_i{}^U(0)) \ge \inf(\alpha_i{}^U(d(x)))\ (\cap\alpha_i{}^U)(d(x))$

$(\cap\beta_i{}^L)(0) = \inf(\beta_i{}^L(0)) \ge \inf(\beta_i{}^L(d(x)))\ (\cap\beta_i{}^L)(d(x))$

$(\cap\beta_i^U)(0) = \inf(\beta_i^U (0)) \geq \inf(\beta_i^U (d(x)))\ (\cap\beta_i^U)(d(x))$

and

$(\cap\alpha_i^L)(d(x*z)) = \inf (\alpha_i^L (d(x * z)) \geq \inf(\min (\alpha_i^L ((d(x)*(y*z)), \alpha_i^L(d(y)))$.

$= \min(\inf (\alpha_i^L ((d(x)*(y*z)), \inf (\alpha_i^L(d(y)))$.

$= \min((\cap\alpha_i^L)((d(x)*(y*z))), ((\cap\alpha_i^L)(d(y)))$

$(\cap\alpha_i^U)(d(x*z)) = \inf (\alpha_i^U (d(x * z)) \geq \inf(\min (\alpha_i^U ((d(x)*(y*z)), \alpha_i^U(d(y)))$.

$= \min(\inf (\alpha_i^U ((d(x)*(y*z)), \inf (\alpha_i^U(d(y)))$.

$= \min((\cap\alpha_i^U)((d(x)*(y*z))), ((\cap\alpha_i^U)(d(y)))$

$(\cap\beta_i^L)(d(x*z)) = \inf (\beta_i^L (d(x * z)) \leq \inf(\max (\beta_i^L ((d(x)*(y*z)), \beta_i^L(d(y)))$.

$= \max(\inf (\beta_i^L ((d(x)*(y*z)), \inf (\beta_i^L(d(y)))$.

$= \max((\cap\beta_i^L)((d(x)*(y*z))), ((\cap\beta_i^L)(d(y)))$

$(\cap\beta_i^U)(d(x*z)) = \inf (\beta_i^U (d(x * z)) \leq \inf(\max (\beta_i^U ((d(x)*(y*z)), \beta_i^U(d(y)))$.

$= \max(\inf (\beta_i^U ((d(x)*(y*z)), \inf (\beta_i^U(d(y)))$.

$= \max((\cap\beta_i^U)((d(x)*(y*z))), ((\cap\beta_i^U)(d(y)))$

This completes the proof .

Lemma 3.8:

The intersection of any set of interval valued intuitionistic fuzzy right derivations KU - ideals of KU–algebra X is also interval valued intuitionistic fuzzy right derivations KU - ideal.

Proof :

The proof is clear.

Theorem 3.9:

Let $((\alpha_A^L , \beta_A^L),(\alpha_A^U , \beta_A^U))$ be a fuzzy set in X then $((\alpha_A^L , \beta_A^L),(\alpha_A^U , \beta_A^U))$ is an interval valued intuitionistic fuzzy left derivations KU- ideal of X if and only if it satisfies :

For all $\gamma \in [0,1]$,$U (((\alpha_A^L , \beta_A^L),(\alpha_A^U , \beta_A^U)) , \gamma) \neq \varphi$ implies $U(((\alpha_A^L , \beta_A^L),(\alpha_A^U , \beta_A^U)) ,\gamma)$ is KU- ideal of X......... (A),

where $U (((\alpha_A^L , \beta_A^L),(\alpha_A^U , \beta_A^U)) , \gamma) = \{x \in X / ((\alpha_A^L , \beta_A^L),(\alpha_A^U , \beta_A^U)) (d(x)) \geq \gamma\}$.

Proof :

Assume that $((\alpha_A^L , \beta_A^L),(\alpha_A^U , \beta_A^U))$ is an interval valued intuitionistic fuzzy left derivations KU- ideal of X , let $\gamma \in [0 , 1]$ be such that $U (((\alpha_A^L , \beta_A^L),(\alpha_A^U , \beta_A^U)) , \gamma) \neq \varphi$, and x , y $\in$X such that $x \in U (((\alpha_A^L , \beta_A^L),(\alpha_A^U , \beta_A^U)) , \gamma)$, then $((\alpha_A^L , \beta_A^L),(\alpha_A^U , \beta_A^U)) (d(x)) \geq \gamma$ and so by

(FL2) , $\alpha_A^L (d(y * 0)) = \alpha_A^L (d(0)) \geq \min \{ \alpha_A^L (d(y)* (x * 0)) , \alpha_A^L (d(x))\}$

$= \min\{ \alpha_A^L (d(y) * 0), \alpha_A^L (d(x))\}$

$= \min \{ \alpha_A^L (0) , \alpha_A^L (d(x))\} = \gamma$,

$\alpha_A^U (d(y * 0)) = \alpha_A^U (d(0)) \geq \min \{ \alpha_A^U (d(y)* (x * 0)) , \alpha_A^U (d(x))\}$

$= \min\{ \alpha_A^U (d(y) * 0), \alpha_A^U (d(x))\}$

$= \min \{ \alpha_A^U (0) , \alpha_A^U (d(x))\} = \gamma$,

$\beta_A^L (d(y * 0)) = \beta_A^L (d(0)) \leq \max \{ \beta_A^L (d(y)* (x * 0)) , \beta^L (d(x))\}$

$= \max\{ \beta_A^L (d(y) * 0), \beta_A^L (d(x))\}$

$= \max \{ \beta_A^L (0) , \beta_A^L (d(x))\} = \gamma$,

$\beta_A^U (d(y * 0)) = \beta_A^U (d(0)) \leq \max \{ \beta_A^U (d(y)* (x * 0)) , \beta_A^U (d(x))\}$

$= \max\{ \beta_A^U (d(y) * 0), \beta_A^U (d(x))\}$

$= \max \{ \beta_A^U (0) , \beta_A^U (d(x))\} = \gamma$

hence $0 \in U (((\alpha_A^L , \beta_A^L),(\alpha_A^U , \beta_A^U)) , \gamma)$.

Let $d(x) * (y * z) \in U (((\alpha_A^L , \beta_A^L),(\alpha_A^U , \beta_A^U)) , \gamma)$, $d(y) \in U (((\alpha_A^L , \beta_A^L),(\alpha_A^U , \beta_A^U)) , \gamma)$, It follows from(IFL3)and(IFL4) that

$\alpha_A^L (d(x * z)) \geq \min \{ \alpha_A^L (d(x) * (y * z)) , \alpha_A^L (d(y))\} = \gamma$,

α_A^U (d(x * z)) ≥ min { α_A^U (d(x) * (y * z)) , α_A^U (d(y))} = γ ,
β_A^L (d(x * z)) ≤ max { β_A^L (d(x) * (y * z)) , β_A^L (d(y))} = γ ,
β_A^U (d(x * z)) ≤ max { β_A^U (d(x) * (y * z)) , β_A^U (d(y))} = γ ,
so that x * z ∈U (((α_A^L , β_A^L),(α_A^U , β_A^U)) , γ) .
Hence U (((α_A^L , β_A^L),(α_A^U , β_A^U)) , γ) is KU - ideal of X .
Conversely, suppose that ((α_A^L , β_A^L),(α_A^U , β_A^U)) satisfies (A) , let x , y , z ∈X be such that
α_A^L(d(x * z)) < min { α_A^L(d(x) * (y * z)) , α_A^L(d(y))},
α_A^U((d(x * z)) < min { α_A^U(d(x) * (y * z)) , α_A^U(d(y))},
β_A^L(d(x * z)) < min { β_A^L (d(x) * (y * z)) , β_A^L (d(y))},
β_A^U(d(x * z)) < min { β_A^U (d(x) * (y * z)) , β_A^U(d(y))},
taking
λ_0 = 1/2 { α_A^L (d(x * z)) + min { α_A^L (d(x) * (y * z)) , α_A^L (d(y)) } ,
λ_0 = 1/2 { α_A^U (d(x * z)) + min { α_A^U (d(x) * (y * z)) , α_A^U (d(y)) },
λ_0 = 1/2 { β_A^L (d(x * z)) + min { β_A^L (d(x) * (y * z)) , β_A^L (d(y)) } ,
λ_0 = 1/2 { β_A^U (d(x * z)) + min { β_A^U (d(x) * (y * z)) , β_A^U (d(y)) }
we have λ_0 ∈ [0,1] and
α_A^L (d(x * z)) < λ_0 < min { α_A^L (d(x) * (y * z)) , α_A^L (d(y)) } ,
α_A^U (d(x * z)) < λ_0 < min { α_A^U (d(x) * (y * z)) , α_A^U (d(y)) } ,
β_A^L (d(x * z)) < λ_0 < min { β_A^L (d(x) * (y * z)) , β_A^L (d(y)) } ,
β_A^U (d(x * z)) < λ_0 < min { β_A^U (d(x) * (y * z)) , β_A^U (d(y)) }
it follows that
d(x) * (y * z) ∈U (((α_A^L,β_A^L),(α_A^U,β_A^U)), λ_0) and d(x * z) ∈U (((α_A^L,β_A^L),(α_A^U,β_A^U)), λ_0) , this is a contradiction and therefore ((α_A^L,β_A^L),(α_A^U,β_A^U)) is an interval valued intuitionistic fuzzy left derivations KU - ideal of X .

Theorem 3.10 :

Let [(α_A^L,β_A^L),(α_A^U,β_A^U)] be an interval valued intuitionistic fuzzy set in X then [(α_A^L,β_A^L),(α_A^U,β_A^U)] is an interval valued intuitionistic fuzzy right derivations KU- ideal of X if and only if it satisfies : For all γ ∈ [0,1] ,U ([(α_A^L,β_A^L),(α_A^U,β_A^U)] , γ) ≠ φ implies U([(α_A^L,β_A^L),(α_A^U,β_A^U)] , γ) is KU- ideal of X.

Proof :

The proof is obvious.

Proposition 3.11 :

If [(α_A^L,β_A^L),(α_A^U,β_A^U)] is an interval valued intuitionistic fuzzy left derivations KU - ideal of X , then
α_A^L (d(x) * (x * y)) ≥ α_A^L (d(y))
α_A^U (d(x) * (x * y)) ≥ α_A^U (d(y))
β_A^L (d(x) * (x * y)) ≤ β_A^L (d(y))
β_A^U (d(x) * (x * y)) ≤ β_A^U (d(y))

proof .

Taking z = x * y in (IFL3) and (IFL4) and using (KU2) and (IFLF1) , we get
α_A^L(d(x) * (x * y)) ≥ min { α_A^L (d(x) * (y * (x * y)) , α_A^L (d(y)) }
= min { α_A^L (d(x) * (x * (y * y)) , α_A^L (d(y)) }
= min { α_A^L (d(x) * (x * 0)) , α_A^L (d(y)) }
= min { α_A^L (0) , α_A^L (d(y)) }
= α_A^L (d(y)).
α_A^U(d(x) * (x * y)) ≥ min { α_A^U (d(x) * (y * (x * y)) , α_A^U (d(y)) }
= min { α_A^U (d(x) * (x * (y * y)) , α_A^U(d(y)) }
= min { α_A^U(d(x) * (x * 0)) , α_A^U(d(y)) }

= min { α_A^U (0) , α_A^U(d(y)) }

= α_A^U(d(y)).

β_A^L(d(x) * (x * y)) ≥ max { β_A^L(d(x) * (y * (x * y)) , β_A^L(d(y))}

= max { β_A^L(d(x) * (x * (y * y)) , β_A^L(d(y))}

= max { β_A^L(d(x) * (x * 0)) , β_A^L(d(y)}

= max { β_A^L(0) , β_A^L(d(y))}

= β_A^L(d(y)).

β_A^U(d(x) * (x * y)) ≥ max { β_A^U(d(x) * (y * (x * y)) , β_A^U(d(y))}

= max { β_A^U(d(x) * (x * (y * y)) , β_A^U(d(y))}

= max { β_A^U(d(x) * (x * 0)) , β_A^U(d(y)}

= max { β_A^U(0) , β_A^U(d(y))}

= β_A^U(d(y)).

Definition 3.12 :

Let ((α_A^L,β_A^L),(α_A^U,β_A^U)) be an interval valued intuitionistic fuzzy left derivations KU - ideal of KU - algebra X , the KU - ideals ((α_t^L,β_t^L),(α_t^U,β_t^U)) , t ∈ [0,1] are called level KU - ideal of ((α_A^L,β_A^L),(α_A^U,β_A^U)).

IV. IMAGE (PRE-IMAGE) OF INTERVAL VALUED INTUITIONISTIC FUZZY DERIVATIONS KU-IDEALS UNDER HOMOMORPHISM

In this section, we introduce the concepts of the image and the pre-image of interval valued intuitionistic fuzzy left derivations KU-ideals in KU-algebras under homomorphism.

Definition 4.1 :

Let f be a mapping from the set X to a set Y. If ((α_A^L,β_A^L),(α_A^U,β_A^U)) is an interval valued intuitionistic fuzzy subset of X, then the interval valued intuitionistic fuzzy subset λ of Y defined by

f((α_A^L,β_A^L),(α_A^U,β_A^U))(y) = λ(y) =

$$\begin{cases} \sup\limits_{x\epsilon f^{-1}(y)} \left((\alpha_A^L,\beta_A^L),(\alpha_A^U,\beta_A^U)\right)(x) & , \ if\ f^{-1}(y)=\{x\in X, f(x)=y\}\neq\emptyset \\ 0 & , \quad otherwise \end{cases}$$

is said to be the image of ((α_A^L,β_A^L),(α_A^U,β_A^U)) under f.

Similarly if λ is an interval valued intuitionistic fuzzy subset of Y , then the interval valued intuitionistic fuzzy subset ((α_A^L,β_A^L),(α_A^U,β_A^U)) = λ o f in X (i.e the interval valued intuitionistic fuzzy subset defined by ((α_A^L,β_A^L),(α_A^U,β_A^U))(x) = λ (f (x)) for all x ∈ X) is called the primage of λ under f .

Theorem 4.1:

An onto homomorphic preimage of an interval valued intuitionistic fuzzy left derivations KU - ideal is also an interval valued intuitionistic fuzzy left derivations KU - ideal .

Proof:

Let f : X → X` be an onto homomorphism of KU - algebras , λ an interval valued intuitionistic fuzzy left derivations KU - ideal of X` and ((α^L,β^L),(α^U,β^U)) the preimage of λ under f , then λ (f (d(x)) = ((α_A^L,β_A^L),(α_A^U,β_A^U))(d(x)) , for all x ∈ X . Let x ∈ X , then

((α_A^L,β_A^L),(α_A^U,β_A^U))(d(0)) = λ (f(d(0))) ≥ λ (f (d(x))) = ((α_A^L,β_A^L),(α_A^U,β_A^U))(d(x)).

Now let x , y , z ∈ X , then

α_A^L (d(x * z)) = λ (f (d(x * z)))

≥ min {λ(f (d(x)) *` (f (y) *` f(z)), λ(f (d(y)))}

= min { λ (f(d(x)*(y * z)),λ (f (d(y)) }

= min { α_A^L(d(x) * (y * z))) , α_A^L(d (y))} .

α_A^U (d(x * z)) = λ (f (d(x * z)))

≥ min {λ(f (d(x)) *` (f (y) *` f(z)), λ(f (d(y)))}

= min { λ (f(d(x)*(y * z)),λ (f (d(y)) }

= min { α_A^U(d(x) * (y * z))) , α_A^U(d (y))} .

β_A^L (d(x * z)) = λ (f (d(x * z)))

≤ max {λ(f (d(x)) *` (f (y) *` f(z)), λ(f (d(y)))}

= max { λ (f(d(x)*(y * z)),λ (f (d(y)) }

= max { β_A^L(d(x) * (y * z))) , β_A^L(d (y))} .

β_A^U(d(x * z)) = λ (f (d(x * z)))

≤ max {λ(f (d(x)) *` (f (y) *` f(z)), λ(f (d(y)))}

= max { λ (f(d(x)*(y * z)),λ (f (d(y)) }

= max { β_A^U(d(x) * (y * z))) , β_A^U(d (y))} .

The proof is completed.

Definition 4.3 :

An interval valued intuitionistic fuzzy subset ((α_A^L,β_A^L),(α_A^U,β_A^U)) of X has sup property if for any subset T of X , there exist t_0 ∈ T such that ,

((α_A^L,β_A^L),(α_A^U,β_A^U))(t_0) = $\sup_{t \in T}$ ((α_A^L,β_A^L),(α_A^U,β_A^U))(t).

Theorem 4.2:

Let f : X → Y be a homomorphism between KU - algebras X and Y . For every interval valued intuitionistic fuzzy left derivations KU - ideal ((α_A^L,β_A^L),(α_A^U,β_A^U)) in X, f ((α_A^L,β_A^L),(α_A^U,β_A^U)) is an interval valued intuitionistic fuzzy left derivations KU - ideal of Y .

Proof:

By definition,

λ(d(y′)) = f ((α_A^L,β_A^L),(α_A^U,β_A^U)) (d(y′)) = $\sup_{d(x) \epsilon f^{-1}(d(y'))}$ ((α_A^L,β_A^L),(α_A^U,β_A^U)) (d(x))

for all y′ ∈ Y and sup ϕ = 0.

We have to prove that

λ (d(x′ * z′)) ≥ min{λ ((d(x′) *(y′* z′)), (d(y′))},

λ (d(x′ * z′)) ≤ max{λ ((d(x′) *(y′* z′)), (d(y′))},for all x′, y′, z′ ∈ Y.

Let f : X → Y be an onto a homomorphism of KU - algebras , ((α_A^L,β_A^L),(α_A^U,β_A^U)) an interval valued intuitionistic fuzzy left derivations KU - ideal of X with sup property and λ the image of ((α_A^L,β_A^L),(α_A^U,β_A^U)) under f ,since ((α_A^L,β_A^L),(α_A^U,β_A^U)) is an interval valued intuitionistic fuzzy left derivations KU - ideal of X , we have

α_A^L (d(0)) ≥ α_A^L (d(x)),

α_A^U (d(0)) ≥ α_A^U (d(x)),

β_A^L (d(0)) ≤ β_A^L (d(x)),

β_A^U (d(0)) ≤ β_A^U (d(x)) for all x ∈ X . Note that 0 ∈ f^{-1}(0′) , where 0 , 0′ are the zero of X and Y respectively.

Thus, (λ(d(0′)) = $\sup_{d(t) \epsilon f^{-1}(d(0'))}$ α_A^L (d(t)) = α_A^L (d(0)) = α_A^L (0) ≥ α_A^L (d(x))

(λ(d(0′)) = $\sup_{d(t) \epsilon f^{-1}(d(0'))}$ α_A^U (d(t)) = α_A^U (d(0)) = α_A^U (0) ≥ α_A^U (d(x))

(λ(d(0′)) = $\sup_{d(t) \epsilon f^{-1}(d(0'))}$ β_A^L (d(t)) = β_A^L (d(0)) = β_A^L (0) ≤ β_A^L (d(x))

(λ(d(0′)) = $\sup_{d(t) \epsilon f^{-1}(d(0'))}$ β_A^U (d(t)) = β_A^U (d(0)) = β_A^U (0) ≤ β_A^U (d(x)) for all x ∈ X ,

which implies that

(λ(d(0′)) ≥ $\sup_{d(t) \epsilon f^{-1}(d(0'))}$ α_A^L (d(t)) = (λ(d(x′))

(λ(d(0′)) ≥ $\sup_{d(t) \epsilon f^{-1}(d(0'))}$ α_A^U (d(t)) = (λ(d(x′))

(λ(d(0′)) ≤ $\sup_{d(t) \epsilon f^{-1}(d(0'))}$ β_A^L (d(t)) = (λ(d(x′))

(λ(d(0′)) ≤ $\sup_{d(t) \epsilon f^{-1}(d(0'))}$ β_A^U (d(t)) = (λ(d(x′)) for any x′ ∈ Y . For any x′, y′,z′ ∈ Y , let

$d(x_0) \in f^{-1}(d(x'))$, $d(y_0) \in f^{-1}(d(y'))$, $d(z_0) \in f^{-1}(d(z'))$ be such that

$\alpha_A^L(d(x_0 * z_0)) = \sup_{d(t)\epsilon f^{-1}(d(x'*z'))} \alpha_A^L(d(t))$, $\alpha_A^L(d(y_0)) = \sup_{d(t)\epsilon f^{-1}(d(y'))} \alpha_A^L(d(t))$

$\alpha_A^U(d(x_0 * z_0)) = \sup_{d(t)\epsilon f^{-1}(d(x'*z'))} \alpha_A^U(d(t))$, $\alpha_A^U(d(y_0)) = \sup_{d(t)\epsilon f^{-1}(d(y'))} \alpha_A^U(d(t))$

$\beta_A^L(d(x_0 * z_0)) = \sup_{d(t)\epsilon f^{-1}(d(x'*z'))} \beta_A^L(d(t))$, $\beta_A^L(d(y_0)) = \sup_{d(t)\epsilon f^{-1}(d(y'))} \beta_A^L(d(t))$

$\beta_A^U(d(x_0 * z_0)) = \sup_{d(t)\epsilon f^{-1}(d(x'*z'))} \beta_A^U(d(t))$, $\beta_A^U(d(y_0)) = \sup_{d(t)\epsilon f^{-1}(d(y'))} \beta_A^U(d(t))$

and

$\alpha_A^L(d(x_0)*(y_0*z_0)) = \lambda\{ f(d(x_0)*(y_0*z_0))\}$

$= \lambda(d(x')*(y'*z'))$

$= \sup_{(d(x_0)*(y_0*z_0)\epsilon f^{-1}(d(x')*(y'*z'))} \alpha_A^L(d(x_0)*(y_0*z_0))$

$= \sup_{(d(t)\epsilon f^{-1}(d(x')*(y'*z'))} \alpha_A^L(d(t))$

$\alpha_A^U(d(x_0)*(y_0*z_0)) = \lambda\{ f(d(x_0)*(y_0*z_0))\}$

$= \lambda(d(x')*(y'*z'))$

$= \sup_{(d(x_0)*(y_0*z_0)\epsilon f^{-1}(d(x')*(y'*z'))} \alpha_A^U(d(x_0)*(y_0*z_0))$

$= \sup_{(d(t)\epsilon f^{-1}(d(x')*(y'*z'))} \alpha_A^U(d(t))$

$\beta_A^L(d(x_0)*(y_0*z_0)) = \lambda\{ f(d(x_0)*(y_0*z_0))\}$

$= \lambda(d(x')*(y'*z'))$

$= \sup_{(d(x_0)*(y_0*z_0)\epsilon f^{-1}(d(x')*(y'*z'))} \beta_A^L(d(x_0)*(y_0*z_0))$

$= \sup_{(d(t)\epsilon f^{-1}(d(x')*(y'*z'))} \beta_A^L(d(t))$

$\beta_A^U(d(x_0)*(y_0*z_0)) = \lambda\{ f(d(x_0)*(y_0*z_0))\}$

$= \lambda(d(x')*(y'*z'))$

$= \sup_{(d(x_0)*(y_0*z_0)\epsilon f^{-1}(d(x')*(y'*z'))} \beta_A^U(d(x_0)*(y_0*z_0))$

$= \sup_{(d(t)\epsilon f^{-1}(d(x')*(y'*z'))} \beta_A^U(d(t))$

Then

$\lambda(d(x' * z')) = \sup_{d(t)\epsilon f^{-1}(d(x'*z'))} \alpha_A^L(d(t))$

$= \alpha_A^L(d(x_0 * z_0))$

$\geq \min\{ \alpha_A^L(d(x_0)*(y_0*z_0)), \alpha_A^L(d(y_0))\}$

$=\min\left\{ \sup_{(d(t)\epsilon f^{-1}(d(x')*(y'*z'))} \alpha_A^L(d(t)), \sup_{d(t)\epsilon f^{-1}(d(y'))} \alpha_A^L(d(t)) \right\}$

$= \min\{\lambda(d(x')*(y'*z')), \lambda(d(y'))\}$

$\lambda(d(x' * z')) = \sup_{d(t)\epsilon f^{-1}(d(x'*z'))} \alpha_A^U(d(t))$

$= \alpha_A^U(d(x_0 * z_0))$

$\geq \min\{ \alpha_A^U(d(x_0)*(y_0*z_0)), \alpha_A^U(d(y_0))\}=$

$\min\left\{ \sup_{(d(t)\epsilon f^{-1}(d(x')*(y'*z'))} \alpha_A^U(d(t)), \sup_{d(t)\epsilon f^{-1}(d(y'))} \alpha_A^U(d(t)) \right\}$

$= \min\{\lambda(d(x')*(y'*z')), \lambda(d(y'))\}$

$\lambda(d(x' * z')) = \sup_{d(t)\epsilon f^{-1}(d(x'*z'))} \beta_A^L(d(t))$

$= \beta_A^L(d(x_0 * z_0))$

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$\leq$ max { $\beta^L(d(x_0)*(y_0*z_0))$, $\beta_A^L(d(y_0))$ }=

$$\max\left\{\sup_{(d(t)\epsilon f^{-1}(d(x')*(y'*z'))}\beta_A^L\left(d(t)\right),\ \sup_{d(t)\epsilon f^{-1}(d(y'))}\beta_A^L\left(d(t)\right)\right\}$$

= max {λ(d (x′)* (y′*z′)) , λ(d(y′))}

$$\lambda\,(d(x' * z')) = \sup_{d(t)\epsilon f^{-1}(d(x'*z'))}\beta_A^U(d(t))$$

= $\beta_A^U(d(x_0 * z_0))$

$\leq$ max { $\beta_A^U(d(x_0)*(y_0*z_0))$, $\beta_A^U(d(y_0))$ }=

$$\max\left\{\sup_{(d(t)\epsilon f^{-1}(d(x')*(y'*z'))}\beta_A^U\left(d(t)\right),\ \sup_{d(t)\epsilon f^{-1}(d(y'))}\beta_A^U\left(d(t)\right)\right\}$$

= max {λ(d (x′)* (y′*z′)) , λ(d(y′))}

Hence λ is an interval valued intuitionistic fuzzy left derivations KU-ideal of Y.

Theorem 4.5:

An onto homomorphic preimage of an interval valued intuitionistic fuzzy right derivations KU - ideal is also an interval valued intuitionistic fuzzy right derivations KU – ideal.

Proof :

The proof is obvious.

Theorem 4.6 :

Let f : X → Y be a homomorphism between KU - algebras X and Y . For every interval valued intuitionistic fuzzy right derivations KU - ideal$((\alpha_A^L,\beta_A^L),(\alpha_A^U,\beta_A^U))$ in X , f $((\alpha_A^L,\beta_A^L),(\alpha_A^U,\beta_A^U))$ is an interval valued intuitionistic fuzzy right derivations KU - ideal of Y .

Proof : The proof is clear.

V. CARTESIAN PRODUCT OF INTERVAL VALUED INTUITIONISTIC FUZZY LEFT DERIVATIONS KU-IDEALS

Definition 5.1 :

An interval valued intuitionistic fuzzy $((\alpha_A^L,\beta_A^L),(\alpha_A^U,\beta_A^U))$ is called an interval valued intuitionistic fuzzy relation on any set S , if $((\alpha^L,\beta^L),(\alpha^U,\beta^U))$ is an interval valued intuitionistic fuzzy subset $((\alpha_A^L,\beta_A^L),(\alpha_A^U,\beta_A^U))$: S × S → [0,1] .

Definition 5.2 :

If$((\alpha_A^L,\beta_A^L),(\alpha_A^U,\beta_A^U))$is an interval valued intuitionistic fuzzy relation on a set S and λ is an interval valued intuitionistic fuzzy subset of S ,then $((\alpha_A^L,\beta_A^L),(\alpha_A^U,\beta_A^U))$ is an interval valued intuitionistic fuzzy relation on λ if $((\alpha_A^L,\beta_A^L),(\alpha_A^U,\beta_A^U))(x , y) \leq \min\{\lambda(x) , \lambda(y)\}$, ∀ x , y ∈ S .

Definition 5.3 :

Let $((\alpha_A^L,\beta_A^L),(\alpha_A^U,\beta_A^U))$ and λ be interval valued intuitionistic fuzzy subset of a set S, the Cartesian product of$((\alpha_A^L,\beta_A^L),(\alpha_A^U,\beta_A^U))$ and λ is define by

$(((\alpha_A^L,\beta_A^L),(\alpha_A^U,\beta_A^U))\times \lambda)\ (x , y) = \min\{((\alpha^L,\beta^L),(\alpha^U,\beta^U))(x) , \lambda\ (y)\}$, ∀ x , y ∈ S .

Lemma 5.4 :

Let $((\alpha_A^L,\beta_A^L),(\alpha_A^U,\beta_A^U))$ and λ be interval valued intuitionistic fuzzy subset of a set S ,then

(i) $(((\alpha_A^L,\beta_A^L),(\alpha_A^U,\beta_A^U))\ x\ \gamma)$ is a fuzzy relation on S .

(ii) $(((\alpha_A^L,\beta_A^L),(\alpha_A^U,\beta_A^U))\ x\ \gamma)_t = ((\alpha_A^L,\beta_A^L),(\alpha_A^U,\beta_A^U))_t\ x\ \gamma_t$ for all t ∈ [0,1].

Definition 5.5 :

If $((\alpha_A^L,\beta_A^L),(\alpha_A^U,\beta_A^U))$ is an interval valued intuitionistic fuzzy left derivations relation on a set S and γ is an interval valued intuitionistic fuzzy left derivations subset of S , then $((\alpha_A^L,\beta_A^L),(\alpha_A^U,\beta_A^U))$ is an interval valued intuitionistic fuzzy left derivations relation on γ if

$(\ ((\alpha_A^L,\beta_A^L),(\alpha_A^U,\beta_A^U))(d\ (x , y)) \leq \min\{\gamma(d\ (x)) , \gamma(d\ (y))\}$, ∀ x , y ∈ S .

Definition 5.6 :

Let $((\alpha_A{}^L,\beta_A{}^L),(\alpha_A{}^U,\beta_A{}^U))$and γ be interval valued intuitionistic fuzzy left derivations subset of a set S,the Cartesian product of $((\alpha_A{}^L,\beta_A{}^L),(\alpha_A{}^U,\beta_A{}^U))$ and γ is define by

$(((\alpha_A{}^L,\beta_A{}^L),(\alpha_A{}^U,\beta_A{}^U)) \times \gamma)(d(x,y)) = \min\{((\alpha_A{}^L,\beta_A{}^L),(\alpha_A{}^U,\beta_A{}^U)))(d(x)), \gamma(d(y))\}, \forall x, y \in S$.

Lemma 5.7:

Let $((\alpha_A{}^L,\beta_A{}^L),(\alpha_A{}^U,\beta_A{}^U))$ and γ be fuzzy subset of a set S ,then

(i) $((\alpha_A{}^L,\beta_A{}^L),(\alpha_A{}^U,\beta_A{}^U)) \times \gamma$ is a fuzzy relation on S ,

(ii) $((\alpha_A{}^L,\beta_A{}^L),(\alpha_A{}^U,\beta_A{}^U)) \times \gamma)_t = ((\alpha_A{}^L,\beta_A{}^L),(\alpha_A{}^U,\beta_A{}^U))_t \times \gamma_t$ for all $t \in [0,1]$.

Definition 5.8 :

If γ is an interval valued intuitionistic fuzzy left derivations subset of a set S , the strongest interval valued intuitionistic fuzzy relation on S , that is an interval valued intuitionistic fuzzy derivations relation on γ is $((\alpha_A{}^L,\beta_A{}^L),(\alpha_A{}^U,\beta_A{}^U))_\gamma$ given by

$((\alpha_A{}^L,\beta_A{}^L),(\alpha_A{}^U,\beta_A{}^U))_\gamma (d(x,y)) = \min\{\gamma(d(x)), \gamma(d(y))\}, \forall x, y \in S$.

Lemma 5.9 :

For a given interval valued intuitionistic fuzzy left derivations subset S , let $((\alpha_A{}^L,\beta_A{}^L),(\alpha_A{}^U,\beta_A{}^U))_\gamma$ be the strongest interval valued intuitionistic fuzzy left derivations relation on S, then for $t \in [0,1]$, we have

$(((\alpha_A{}^L,\beta_A{}^L),(\alpha_A{}^U,\beta_A{}^U))\gamma)_t = \gamma_t \times \gamma_t$.

Proposition5.10 :

For a given interval valued intuitionistic fuzzy subset γ of KU - algebra X , let $((\alpha_A{}^L,\beta_A{}^L),(\alpha_A{}^U,\beta_A{}^U))_\gamma$ be the strongest left interval valued intuitionistic fuzzy derivations relation on X . If $((\alpha_A{}^L,\beta_A{}^L),(\alpha_A{}^U,\beta_A{}^U))_\gamma$ is an interval valued intuitionistic fuzzy left derivations KU - ideal of X × X , then $\gamma(d(x)) \le \gamma(d(0)) = \gamma(0)$ for all $x \in X$.

Proof :

Since $((\alpha_A{}^L,\beta_A{}^L),(\alpha_A{}^U,\beta_A{}^U))\gamma$ is an interval valued intuitionistic fuzzy left derivations KU- ideal of X × X , it follows from (F1) that

$\alpha_A{}^L{}_\gamma(x,x) = \min\{\gamma(d(x)), \gamma(d(x))\} \le \gamma(d(0,0)) = \min\{\gamma(d(0)), \gamma(d(0))\}$,

$\alpha_A{}^U{}_\gamma(x,x) = \min\{\gamma(d(x)), \gamma(d(x))\} \le \gamma(d(0,0)) = \min\{\gamma(d(0)), \gamma(d(0))\}$,

$\beta_A{}^L{}_\gamma(x,x) = \min\{\gamma(d(x)), \gamma(d(x))\} \le \gamma(d(0,0)) = \min\{\gamma(d(0)), \gamma(d(0))\}$,

$\beta_A{}^U{}_\gamma(x,x) = \min\{\gamma(d(x)), \gamma(d(x))\} \le \gamma(d(0,0)) = \min\{\gamma(d(0)), \gamma(d(0))\}$,

where $(0,0) \in X \times X$ then $\gamma(d(x)) \le \gamma(d(0)) = \gamma(0)$.

Remark 5.11:

Let X and Y be KU- algebras , we define * on X × Y by :

For every $(x,y), (u,v) \in X \times Y$, $(x,y) * (u,v) = (x * u, y * v)$, then clearly

$(X \times Y, *, (0,0))$ is a KU- algebra .

Theorem 5.12 :

Let $((\alpha_A{}^L,\beta_A{}^L),(\alpha_A{}^U,\beta_A{}^U))$ and γ be an interval valued intuitionistic fuzzy left derivations KU- ideals of KU - algebra X ,then $((\alpha_A{}^L,\beta_A{}^L),(\alpha_A{}^U,\beta_A{}^U)) \times \gamma$ is an interval valued intuitionistic fuzzy left derivations KU-ideal of X × X .

Proof :

For any $(x,y) \in X \times X$,we have ,

$(\alpha_A{}^L \times \gamma)(d((0,0)) = \min\{\alpha_A{}^L(d(0)), \gamma(d(0))\} = \min\{\alpha_A{}^L(0), \gamma(0)\}$

$\ge \min\{\alpha_A{}^L(d(x)), \gamma(d(x))\} = (\alpha_A{}^L \times \gamma)(d(x,y))$.

$(\alpha_A{}^U \times \gamma)(d((0,0)) = \min\{\alpha_A{}^U(d(0)), \gamma(d(0))\} = \min\{\alpha_A{}^U(0), \gamma(0)\}$

$\ge \min\{\alpha_A{}^U(d(x)), \gamma(d(x))\} = (\alpha_A{}^U \times \gamma)(d(x,y))$.

$(\beta_A{}^L \times \gamma)(d((0,0)) = \max\{\beta_A{}^L(d(0)), \gamma(d(0))\} = \max\{\beta_A{}^L(0), \gamma(0)\}$

$\le \max\{\beta_A{}^L(d(x)), \gamma(d(x))\} = (\beta_A{}^L \times \gamma)(d(x,y))$.

$(\beta_A{}^U \times \gamma)(d((0,0)) = \max\{\beta_A{}^U(d(0)), \gamma(d(0))\} = \max\{\beta_A{}^U(0), \gamma(0)\}$

$\le \max\{\beta_A{}^U(d(x)), \gamma(d(x))\} = (\beta_A{}^U \times \gamma)(d(x,y))$.

Now let $(x_1, x_2), (y_1, y_2), (z_1, z_2) \in X \times X$, then ,

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$(\alpha_A^L \times \gamma)(d(x_1 * z_1, x_2 * z_2))$
$= \min\{\alpha_A^L(d(x_1, z_1)), \gamma(d(x_2, z_2))\}$
$\geq \min\{\min\{\alpha_A^L(d(x_1)*(y_1 * z_1))), \alpha_A^L(d(y_1))\}\}, \min\{\gamma(d(x_2)*(y_2 * z_2))), \gamma(d(y_2))\}\}$
$= \min\{\min\{\alpha_A^L(d(x_1)*(y_1 * z_1))), \alpha_A^L(d(x_2)*(y_2 * z_2)))\}, \min\{\alpha_A^L(d(y_1)), \gamma(d(y_2))\}\}$
$= \min\{(\alpha_A^L \times \gamma)((d(x_1)*(y_1 * z_1), d(x_2)*(y_2 * z_2))), (\alpha_A^L \times \gamma)(d(y_1), d(y_2))\}$.
$(\alpha_A^U \times \gamma)(d(x_1 * z_1, x_2 * z_2))$
$= \min\{\alpha_A^U(d(x_1, z_1)), \gamma(d(x_2, z_2))\}$
$\geq \min\{\min\{\alpha_A^U(d(x_1)*(y_1 * z_1))), \alpha_A^L(d(y_1))\}\}, \min\{\gamma(d(x_2)*(y_2 * z_2))), \gamma(d(y_2))\}\}$
$= \min\{\min\{\alpha_A^U(d(x_1)*(y_1 * z_1))), \alpha_A^L(d(x_2)*(y_2 * z_2)))\}, \min\{\alpha_A^U(d(y_1)), \gamma(d(y_2))\}\}$
$= \min\{(\alpha_A^U \times \gamma)((d(x_1)*(y_1 * z_1), d(x_2)*(y_2 * z_2))), (\alpha_A^U \times \gamma)(d(y_1), d(y_2))\}$
$(\beta_A^L \times \gamma)(d(x_1 * z_1, x_2 * z_2))$
$= \max\{\beta_A^L(d(x_1, z_1)), \gamma(d(x_2, z_2))\}$
$\leq \max\{\max\{\beta_A^L(d(x_1)*(y_1 * z_1))), \beta_A^L(d(y_1))\}\}, \max\{\gamma(d(x_2)*(y_2 * z_2))), \gamma(d(y_2))\}\}$
$= \max\{\max\{\beta_A^L(d(x_1)*(y_1 * z_1))), \beta_A^L(d(x_2)*(y_2 * z_2)))\}, \max\{\beta_A^L(d(y_1)), \gamma(d(y_2))\}\}$
$= \max\{(\beta_A^L \times \gamma)((d(x_1)*(y_1 * z_1), d(x_2)*(y_2 * z_2))), (\beta_A^L \times \gamma)(d(y_1), d(y_2))\}$
$(\beta_A^U \times \gamma)(d(x_1 * z_1, x_2 * z_2))$
$= \max\{\beta_A^U(d(x_1, z_1)), \gamma(d(x_2, z_2))\}$
$\leq \max\{\max\{\beta_A^U(d(x_1)*(y_1 * z_1))), \beta_A^U(d(y_1))\}\}, \max\{\gamma(d(x_2)*(y_2 * z_2))), \gamma(d(y_2))\}\}$
$= \max\{\max\{\beta_A^U(d(x_1)*(y_1 * z_1))), \beta_A^U(d(x_2)*(y_2 * z_2)))\}, \max\{\beta_A^U(d(y_1)), \gamma(d(y_2))\}\}$
$= \max\{(\beta_A^U \times \gamma)((d(x_1)*(y_1 * z_1), d(x_2)*(y_2 * z_2))), (\beta_A^U \times \gamma)(d(y_1), d(y_2))\}$
Hence $((\alpha_A^L,\beta_A^L),(\alpha_A^U,\beta_A^U)) \times \gamma$ is a fuzzy left derivations KU- ideal of $X \times X$.

Remark 5.13:

Let$((\alpha_A^L,\beta_A^L),(\alpha_A^U,\beta_A^U))$ and γ be an interval valued intuitionistic fuzzy left derivations subset of KU-algebra X ,such that $((\alpha_A^L,\beta_A^L),(\alpha_A^U,\beta_A^U)) \times \gamma$ is an interval valued intuitionistic fuzzy left derivations KU-ideal of $X \times X$, then

(i)Either $((\alpha_A^L,\beta_A^L),(\alpha_A^U,\beta_A^U))(d(x)) \leq ((\alpha_A^L,\beta_A^L),(\alpha_A^U,\beta_A^U))(d(0))$ or $\gamma(d(x)) \leq \gamma(d(0))$ for all $x \in X$.

(ii) If $((\alpha_A^L,\beta_A^L),(\alpha_A^U,\beta_A^U))(d(x)) \leq ((\alpha_A^L,\beta_A^L),(\alpha_A^U,\beta_A^U))(d(0))$ for all $x \in X$, then either $((\alpha_A^L,\beta_A^L),(\alpha_A^U,\beta_A^U))(d(x)) \leq \gamma(d(0))$ or $\gamma(d(x)) \leq \gamma(d(0))$,

(iii) If $\gamma(d(x)) \leq \gamma(d(0))$ for all $x \in X$, then either $((\alpha_A^L,\beta_A^L),(\alpha_A^U,\beta_A^U))(d(x)) \leq ((\alpha_A^L,\beta_A^L),(\alpha_A^U,\beta_A^U))(d(0))$ or $\gamma(d(x)) \leq ((\alpha_A^L,\beta_A^L),(\alpha_A^U,\beta_A^U))(d(0))$,

(iv) Either $((\alpha_A^L,\beta_A^L),(\alpha_A^U,\beta_A^U))$ or γ is an interval valurd intuitionistic fuzzy left derivations KU- ideal of X .

Theorem 5.14 :

Let γ be an interval valued intuitionistic fuzzy subset of KU- algebra X and let $((\alpha_A^L,\beta_A^L),(\alpha_A^U,\beta_A^U))_\gamma$ be the strongest interval valued intuitionistic fuzzy left derivations relation on X , then γ is an interval valued intuitionistic fuzzy left derivations KU - ideal of X if and only if $((\alpha_A^L,\beta_A^L),(\alpha_A^U,\beta_A^U))_\gamma$ is an interval valued intuitionistic fuzzy left derivations KU- ideal of $X \times X$

proof :

Assume that γ is an interval valued intuitionistic fuzzy left derivations KU- ideal X , we note from (F1) that :
$\alpha_A{}^L{}_\gamma(0, 0) = \min\{\gamma(d(0)), \gamma(d(0))\} = \min\{\gamma(0), \gamma(0)\} \geq \min\{\gamma(d(x)), \gamma(d(y))\}$
$= \alpha_A{}^L{}_\gamma(d(x), d(y))$.
$\alpha_A{}^U{}_\gamma(0, 0) = \min\{\gamma(d(0)), \gamma(d(0))\} = \min\{\gamma(0), \gamma(0)\} \geq \min\{\gamma(d(x)), \gamma(d(y))\}$
$= \alpha_A{}^U{}_\gamma(d(x), d(y))$.
$\beta_A{}^L{}_\gamma(0, 0) = \min\{\gamma(d(0)), \gamma(d(0))\} = \min\{\gamma(0), \gamma(0)\} \geq \min\{\gamma(d(x)), \gamma(d(y))\}$
$= \beta_A{}^L{}_\gamma(d(x), d(y))$.
$\beta_A{}^U{}_\gamma(0, 0) = \min\{\gamma(d(0)), \gamma(d(0))\} = \min\{\gamma(0), \gamma(0)\} \geq \min\{\gamma(d(x)), \gamma(d(y))\}$
$= \beta_A{}^U{}_\gamma(d(x), d(y))$.

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Now, for any (x_1,x_2) , (y_1,y_2) ,$(z_1,z_2) \in X \times X$, we have from (F2) :

$\alpha_A^{L}{}_{\gamma} (d(x_1 * z_1) , d(x_2 * z_2))$

$= \min \{ \gamma (d(x_1 * z_1)) , \gamma (d(x_2 * z_2))\}$

$\geq \min \{\min\{ \gamma (d (x_1)*(y_1 * z_1)) , \gamma (d(y_1))\} , \min \{ \gamma (d(x_2) * (y_2 * z_2)) , \gamma (y_2)\}\}$

$= \min\{\min\{ \gamma (d(x_1) * (y_1 * z_1)) , \gamma (d(x_2) * (y_2 * z_2))\} , \min\{ \gamma (d(y_1)), \gamma (d(y_2))\}\}$

$= \min \{ \alpha_A^{L}{}_{\gamma} (d(x_1) * (y_1 * z_1) , d(x_2) * (y_2 * z_2)) , \alpha_A^{L}{}_{\gamma}(d(y_1) , d(y_2))\}$.

$\alpha_A^{U}{}_{\gamma} (d(x_1 * z_1) , d(x_2 * z_2))$

$= \min \{ \gamma (d(x_1 * z_1)) , \gamma (d(x_2 * z_2))\}$

$\geq \min \{\min\{ \gamma (d (x_1)*(y_1 * z_1)) , \gamma (d(y_1))\} , \min \{ \gamma (d(x_2) * (y_2 * z_2)) , \gamma (y_2)\}\}$

$= \min\{\min\{ \gamma (d(x_1) * (y_1 * z_1)) , \gamma (d(x_2) * (y_2 * z_2))\} , \min\{ \gamma (d(y_1)), \gamma (d(y_2))\}\}$

$= \min \{ \alpha_A^{U}{}_{\gamma} (d(x_1) * (y_1 * z_1) , d(x_2) * (y_2 * z_2)) , \alpha_A^{U}{}_{\gamma}(d(y_1) , d(y_2))\}$.

$\beta_A^{L}{}_{\gamma} (d(x_1 * z_1) , d(x_2 * z_2))$

$= \max \{ \gamma (d(x_1 * z_1)) , \gamma (d(x_2 * z_2))\}$

$\leq \max \{\max\{ \gamma (d (x_1)*(y_1 * z_1)) , \gamma (d(y_1))\} , \max \{ \gamma (d(x_2) * (y_2 * z_2)) , \gamma (y_2)\}\}$

$= \max\{\max\{ \gamma(d(x_1) * (y_1 * z_1)) ,\gamma (d(x_2) * (y_2 * z_2))\} , \max \{ \gamma (d(y_1)), \gamma (d(y_2))\}\}$

$= \max \{ \beta_A^{L}{}_{\gamma} (d(x_1) * (y_1 * z_1) , d(x_2) * (y_2 * z_2)) , \beta_A^{U}{}_{\gamma}(d(y_1) , d(y_2))\}$.

$\beta_A^{U}{}_{\gamma} (d(x_1 * z_1) , d(x_2 * z_2))$

$= \max \{ \gamma (d(x_1 * z_1)) , \gamma (d(x_2 * z_2))\}$

$\leq \max \{\max\{ \gamma (d (x_1)*(y_1 * z_1)) , \gamma (d(y_1))\} , \max \{ \gamma (d(x_2) * (y_2 * z_2)) , \gamma (y_2)\}\}$

$= \max\{\max\{ \gamma(d(x_1) * (y_1 * z_1)) , \gamma (d(x_2) *(y_2 * z_2))\} , \max \{ \gamma (d(y_1)), \gamma (d(y_2))\}\}$

$= \max \{ \beta_A^{U}{}_{\gamma} (d(x_1) * (y_1 * z_1) , d(x_2) * (y_2 * z_2)) , \beta_A^{U}{}_{\gamma}(d(y_1) , d(y_2))\}$.

Hence $((\alpha_A^{L},\beta_A^{L}),(\alpha_A^{U},\beta_A^{U}))_{\gamma}$ is an interval valued intuitionistic fuzzy left derivations KU - ideal of $X \times X$.

Conversely . For all $(x , y) \in X \times X$, we have

$\min \{ \gamma (0) , \gamma (0) \} = ((\alpha_A^{L},\beta_A^{L}),(\alpha_A^{U},\beta_A^{U}))_{\gamma}(x , y) = \min \{ \gamma (x) , \gamma (y)\}$. It follows that

$\gamma (0) \geq \gamma (x)$ for all $x \in X$, which prove (F1).

Now, let (x_1 , x_2) , (y_1 , y_2), $(z_1 , z_2) \in X \times X$, then

$\min \{ \gamma (d(x_1 * z_1)) , \gamma d(x_2 * z_2))\}$

$= \alpha_A^{L}{}_{\gamma} (d(x_1 * z_1) , d(x_2 * z_2))$

$\geq \min \{ \alpha_A^{L}{}_{\gamma} (d(x_1 , x_2) * ((y_1 , y_2) * (z_1, z_2)) , \alpha_A^{L}{}_{\gamma} (d(y_1),d(y_2))\}$

$= \min \{ \alpha_A^{L}{}_{\gamma} (d(x_1) * (y_1 * z_1) , d(x_2) * (y_2 * z_2)) , \alpha_A^{L}{}_{\gamma} (d(y_1) , d(y_2))\}$

$= \min \{\min \{ \gamma (d(x_1)* (y_1 * z_1)) , \gamma (d(x_2) *(y_2 * z_2))\} , \min \{ \gamma (d(y_1)) , \gamma (d(y_2))\}\}$

$= \min \{\min \{ \gamma (d(x_1) *(y_1 * z_1)) , \gamma (d(y_1))\} , \min \{ \gamma ((d(x_2) * (y_2 * z_2)) , \gamma(d(y_2))\}\}$

$\min \{ \gamma (d(x_1 * z_1)) , \gamma d(x_2 * z_2))\}$

$= \alpha_A^{U}{}_{\gamma} (d(x_1 * z_1) , d(x_2 * z_2))$

$\geq \min \{ \alpha_A^{U}{}_{\gamma} (d(x_1 , x_2) * ((y_1 , y_2) * (z_1, z_2)) , \alpha_A^{L}{}_{\gamma} (d(y_1),d(y_2))\}$

$= \min \{ \alpha^{U}{}_{\gamma} (d(x_1) * (y_1 * z_1) , d(x_2) * (y_2 * z_2)) , \alpha_A^{L}{}_{\gamma} (d(y_1) , d(y_2))\}$

$= \min \{\min \{ \gamma (d(x_1)*(y_1 * z_1)) , \gamma (d(x_2) * (y_2 * z_2))\} , \min \{ \gamma (d(y_1)) , \gamma (d(y_2))\}\}$

$= \min \{\min \{ \gamma (d(x_1) *(y_1 * z_1)) , \gamma (d(y_1))\} , \min \{ \gamma ((d(x_2) * (y_2 * z_2)) , \gamma(d(y_2))\}\}$

$\max \{ \gamma (d(x_1 * z_1)) , \gamma d(x_2 * z_2))\}$

$= \beta_A^{L}{}_{\gamma} (d(x_1 * z_1) , d(x_2 * z_2))$

$\leq \max \{ \beta_A^{L}{}_{\gamma} (d(x_1 , x_2) * ((y_1 , y_2) * (z_1, z_2)) , \alpha_A^{L}{}_{\gamma} (d(y_1),d(y_2))\}$

$= \max \{ \beta_A^{L}{}_{\gamma} (d(x_1) * (y_1 * z_1) , d(x_2) * (y_2 * z_2)) , \alpha_A^{L}{}_{\gamma} (d(y_1) , d(y_2))\}$

$= \max \{\max\{ \gamma(d(x_1)* (y_1 * z_1)) ,\gamma (d(x_2) *(y_2 * z_2))\} , \max \{ \gamma (d(y_1)) , \gamma (d(y_2))\}\}$

$= \max \{\max\{ \gamma (d(x_1) *(y_1 * z_1)) ,\gamma (d(y_1))\} , \max\{ \gamma ((d(x_2) * (y_2 * z_2)) , \gamma(d(y_2))\}\}$

$\max \{ \gamma (d(x_1 * z_1)) , \gamma d(x_2 * z_2))\}$

$= \beta_A{}^L{}_\gamma (d(x_1 * z_1), d(x_2 * z_2))$

$\leq \max \{ \beta_A{}^L{}_\gamma (d(x_1, x_2) * ((y_1, y_2) * (z_1, z_2)), \alpha_A{}^L{}_\gamma (d(y_1), d(y_2))\}$

$= \max \{ \beta_A{}^L{}_\gamma (d(x_1) * (y_1 * z_1), d(x_2) * (y_2 * z_2)), \alpha_A{}^L{}_\gamma (d(y_1), d(y_2))\}$

$= \max \{\max\{ \gamma(d(x_1)* (y_1 * z_1)), \gamma (d(x_2) *(y_2 * z_2))\}, \max \{ \gamma (d(y_1)), \gamma (d(y_2))\}\}$

$= \max \{\max\{ \gamma (d(x_1) *(y_1 * z_1)), \gamma (d(y_1))\}, \max\{ \gamma ((d(x_2) * (y_2 * z_2)), \gamma(d(y_2))\}\}$

In particular, if we take $x_2 = y_2 = z_2 = 0$, then ,

$\gamma (d(x_1 * z_1)) \geq \min \{ \gamma (d(x_1) * (y_1 * z_1)), \gamma (d(y_1))\}$

$\gamma (d(x_1 * z_1)) \leq \max \{ \gamma (d(x_1) * (y_1 * z_1)), \gamma (d(y_1))\}$

This prove (IFL3) and (IFL4).

This completes the proof.

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