e-ISSN: 2582-5208
International Research Journal of Modernization in Engineering Technology and Science
(Peer-Reviewed, Open Access, Fully Refereed International Journal)
Volume:07/Issue:03/March-2025 Impact Factor- 8.187 www.irjmets.com

INTUITIONISTIC FUZZY NANO*GENERALIZED CLOSED SETS AND CONTINUOUS MAPPINGS

J. Jelsiya Josis*1, Dr. M. Trinita Pricilla*2

*1PG Student, Department Of Mathematics, Nirmala College For Women, Coimbatore, Tamil Nadu, India.

*2Assistant Professor, Department Of Mathematics, Nirmala College For Women, Coimbatore, Tamil Nadu, India.

DOI: https://www.doi.org/10.56726/IRJMETS68603

ABSTRACT

In this paper we have introduced intuitionistic fuzzy nano*generalized closed sets and intuitionistic fuzzy nano*generalized continuous mappings. Also, their characterizations are analyzed. We have compared the set with the other existing sets.

Keywords: Intuitionistic Fuzzy Nano Topological Space; Intuitionistic Fuzzy Nano*Generalized Closed Sets; Intuitionistic Fuzzy Nano*Generalized Continuous Mappings; Intuitionistic Fuzzy Nano Closed Set.

I. INTRODUCTION

The concept of fuzzy sets (FS) was first introduced by Zadeh [13] in 1965 as a means to handle uncertainty, vagueness, and partial truth by assigning a membership degree to each element within a given universal set. Later, in 1983, Atanassov [1] extended this idea by incorporating a degree of non-membership, leading to the development of intuitionistic fuzzy sets (IFS). This extension provides a more refined approach to uncertainty quantification, allowing for a more precise representation of problems based on available knowledge and observations.

The foundation of fuzzy topology was laid by Chang in 1967, after which numerous generalizations of fuzzy sets and fuzzy topology emerged. Over the past few years, many of these concepts have been further extended to intuitionistic fuzzy sets. In 1997, Coker [3] introduced the notion of an intuitionistic fuzzy topological space using intuitionistic fuzzy sets.

In this study, we present the concepts of intuitionistic fuzzy nano*generalized closed sets and intuitionistic fuzzy nano*generalized continuous mappings. Furthermore, we explore and analysed their fundamental properties and characterizations.

II. PRELIMINARIES

Definition 2.1 [6]

Let us consider U be the universal set, R be an equivalence relation on U. Define $\tau'_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. The collection, $\tau'_R(X)$ satisfies the following

axioms:

(1) Both U and ϕ belongs to $\tau'_R(X)$.

(2) The union of any subcollection of $\tau'_R(X)$ is also in $\tau'_R(X)$.

(3) The intersection of any finite subcollection of $\tau'_R(X)$ remains in $\tau'_R(X)$.

Thus, $\tau'_R(X)$ forms a topology on U, known as the nano topology on U with respect to X. The pair $(U, \tau'_R(X))$ is called as the nano topological space and the elements of $\tau'_R(X)$ are referred to as nano-open sets. For a nano topological space $(U,\tau'_R(X))$ where $X \subseteq U$ and a subset $A \subseteq U$, the nano interior of A is defined as the union of all nano-open subsets of A and it is denoted by NInt(A). NInt(A) is the largest nano-open subset of A. Similarly, the nano closure of A is defined as the intersection of all nano closed sets containing A and it is denoted by NCl(A). This means NCl(A) is the smallest nano closed set that containing A.

Definition 2.2 [6]

Consider $(U, \tau'_R(X))$ and $(V, \tau''_R(Y))$ be two nano topological spaces. A function $f: (U, \tau'_R(X)) \longrightarrow (V, \tau''_R(Y))$ is called nano continuous on U if the inverse image of every nano-open set in V is also a nano-open set in U.

Definition 2.3 [2]

Consider (U, $\tau'_R(X)$) as a nano topological space. A subset A of (U, $\tau'_R(X)$) is said to be nano generalized closed set (abbreviated as Ng-closed) if NCl(A) ⊆ V where A ⊆ V and V is nano- open set.

Definition 2.4 [8]

Let U be a non-empty finite set, referred to as the universe and R be an intuitionistic fuzzy equivalence relation on U known as the indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is termed as an intuitionistic fuzzy approximation space (IFAS). For any subset X ⊆ U,

1. **Lower Approximation:** The lower approximation of X with respect to R is the set of all objects that can be definitively classified as elements of X under R. It is denoted by $IFL_R(X)$ and is given by: $IFL_R(X) = U_{x\epsilon U}$ {R(x) such that R(x) ⊆ X} where R(x) represents the equivalence class determined by x ∈ U.
2. **Upper Approximation:** The upper approximation of X under R consist of all objects, which can be possibly classified as X with respect to R and it is denoted by $IFU_R(X)$ and is given by: $IFU_R(X)= U_{x\epsilon U}$\{R(x) such that R(x) ∩ X ≠ ϕ\}.
3. **Boundary Region:** The boundary region of X with respect to R consist of objects that cannot be conclusively classified as either belonging to X or not. It is defined as $IFB_R(X)$. That is, $IFB_R(X)=IFU_R(X)-IFL_R(X)$.

Now let U be the universe and R be an intuitionistic fuzzy equivalence relation on U. Define $\tau'_R(X)$= {1, 0, $IFL_R(X)$, $IFU_R(X)$, $IFB_R(X)$} where X ⊆ U. Then $\tau'_R(X)$ satisfies the following axioms:

(1) Both 1 and 0 ∈ $\tau'_R(X)$.

(2) The union of any subcollection of $\tau'_R(X)$ is in $\tau'_R(X)$.

(3) The intersection of any finite subcollection of $\tau'_R(X)$ is in $\tau'_R(X)$.

Therfore $\tau'_R(X)$ forms a topology on U referred to as the intuitionistic fuzzy nano topology on U with respect to X. The pair (U, $\tau'_R(X)$) is known as the intuitionistic fuzzy nano topological space and the elements of $\tau'_R(X)$ are called as intuitionistic fuzzy nano-open sets. If (U, $\tau'_R(X)$) is a intuitionistic fuzzy nano topological space (In short IFNTS) where X⊆U and if A⊆U, then the intuitionistic fuzzy nano interior of A is defined as the union of all intuitionistic fuzzy nano-open subsets of A and it is denoted by IFNInt(A). IFNInt(A) is the largest intuitionistic fuzzy nano-open subset of A. The intuitionistic fuzzy nano closure of A is defined as the intersection of all intuitionistic fuzzy nano closed sets (IFNCS) containing A and it is denoted by IFNCl(A). That is, IFNCl(A) is the smallest intuitionistic fuzzy nano closed set containing A.

Definition 2.5 [1]

An intuitionistic fuzzy set (IFS) A is defined as a collection of elements in the form A = {⟨x, $\mu_A(x)$, $\nu_A(x)$⟩ such that x ∈ X}. Here the functions μ_A: X → [0,1] and ν_A: X → [0,1] represents the degree of membership ($\mu_A(x)$) and the degree of non-membership ($\nu_A(x)$) of each element x ∈ X to the set A, respectively and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each x ∈ X. The collection of all (X), the set of all intuitionistic fuzzy sets in X.

For simplicity, an intuitionistic fuzzy set A in X can also be expressed as A=⟨x, μ_A, ν_A⟩ instead of writing it in the expanded form of A= {⟨x, $\mu_A(x)$, $\nu_A(x)$⟩ such that x ∈ X}.

Definition 2.6 [1]

Consider two intuitionistic fuzzy sets A and B defined as A = {⟨x, $\mu_A(x)$, $\nu_A(x)$⟩ such that x ∈ X} and B = {⟨x, $\mu_B(x)$, $\nu_B(x)$⟩ such that x ∈ X}. Then the following properties hold:

(a) A⊆B if and only if for every x ∈ X, the membership degree of A does not exceed of B ($\mu_A(x) \leq \mu_B(x)$) and the membership degree of A is at least the non-membership degree of B ($\mu_A(x) \geq \nu_B(x)$) for all x ∈ X.

(b) A = B if and only if both A ⊆ B and A ⊇ B hold.

(c) The complement of A is given by A^c = {⟨x, $\nu_A(x)$, $\mu_A(x)$⟩ such that x ∈ X}, where the membership and non-membership values are swapped.

(d) The union of A and B is represented as A∪B = {⟨x, max($\mu_A(x)$, $\mu_B(x)$), min($\nu_A(x)$, $\nu_B(x)$) such that x ∈ X}.

(e) The intersection of A and B is defined as A∩B = {⟨x, min($\mu_A(x)$, $\mu_B(x)$), max($\nu_A(x)$,$\nu_B(x)$)⟩ such that x ∈ X}.

Additionally, the intuitionistic fuzzy sets 0 = ⟨x, 0, 1⟩ represents the empty set, while 1 = ⟨x, 1, 0⟩ corresponds to the universal set X.

Definition 2.7 [3]

An intuitionistic fuzzy topology (IFT) on a set X is a collection τ of IFSs in X that satisfying the following conditions:

(i) 0,1 ∈ τ

(ii) G1∩G2 ∈ τ for any G1, G2 ∈ τ

(iii) ∪G_i ∈ τ for any family {G_i such that i ∈ J} ⊆ τ

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X. The complement A^c of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition 2.8 [12]

Two intuitionistic fuzzy sets (IFS) are considered to be q-coincident (denoted as $A_q B$) if and only if there exists at least one element x ∈ X such that $\mu_A(x) > \nu_B(x)$ or $\nu_A(x) < \mu_B(x)$.

Definition 2.9 [5]

Let A be an intuitionistic fuzzy set in an intuitionistic fuzzy topological space (X, τ). Then the β-interior and β-closure of A are defined as follows:

βint(A) = ∪ {G such that G is an IFβOS in X and G ⊆ A},

βcl(A) = ∩ {K such that K is an IFβCS in X and A ⊆ K}.

Additionally, for any IFS A in (X, τ), the following relationships hold:

βcl(A^c) = (βint(A))^c and βint(A^c) = (βcl(A))^c.

Definition 2.10 [9]

An intuitionistic fuzzy set A in an IFTS (X, τ) is said to be an intuitionistic fuzzy β generalized closed set (IFβGCS for short) if βcl(A)⊆U whenever A⊆U and U is an IFβOS in (X, τ).

The complement A^c of an IFβGCS A in an IFTS (X, τ) is called an intuitionistic fuzzy β generalized open set (IFβGOS in short) in X.

Definition 2.11 [4]

Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an intuitionistic fuzzy continuous (IF continuous) mapping if $f^{-1}(B)$ ∈ IFO(X) for every B ∈ σ.

III. INTUITIONISTIC FUZZY NANO*GENERALIZED CLOSED SET

Throughout this paper (U,τ'_R(X)) is an intuitionistic fuzzy nano topological space with respect to X where X ⊆ U, R is an equivalence relation on U, U|R denotes the family of equivalence classes of U by R.

Definition 3.1

Let (U,τ'_R(X)) be an intuitionistic fuzzy nano topological space. A subset A of (U,τ'_R(X)) is called intuitionistic fuzzy nano*generalized closed set (briefly IFN*g-closed) if IFNCl(A) ⊆ V where A ⊆ V and V is intuitionistic fuzzy nano generalized open (IFNgOS).

Theorem 3.2

A subset B of (U,τ'_R(X)) is IFN*g-closed if IFNCl(B)−B contains no nonempty IFNg-closed set.

Proof:

Suppose if B is IFN*g-closed. Then IFNCl(B) ⊆ V where B ⊆ V and V is IFNgO. Let Y be an intuitionistic fuzzy nano generalized closed subset of IFNCl(B) −B. Then B⊆$Y^c$and Y^c is IFNgO. Since B is N*g-closed, IFNCl(B) ⊆ Y^c or Y ⊆ (IFNCl(B))^c. That is Y ⊆ IFNCl(B) and Y ⊆ (IFNCl(B))^c implies that Y ⊆ ϕ. So, Y is empty.

Theorem 3.3

If A and B are IFN*g-closed then A∪B is IFN*g-closed.

Proof:

Let A and B be IFN*g-closed sets. Then IFNCl(A) ⊆ V where A ⊆ V and V is IFNgO and IFNCl(B) ⊆ V where B ⊆ V and V is IFNgOS. Since A and B are subsets of V, A∪B is a subset of V and V is IFNgOS. Then IFNCl(A∪B) = IFNCl(A) ∪ IFNCl(B) ⊆ V which implies that A∪B is IFN*g-closed.

Theorem 3.4

If C is IFN*g-closed and C ⊆ D ⊆ IFNCl(C) then D is IFN*g-closed.

Proof:

Let D ⊆ V where V is IFNgOS in $\tau_R'(X)$. Then C ⊆ D implies C ⊆ V. Since C in IFN*g-closed, IFNCl(C)⊆V. Also, C⊆NCl(D) implies IFNCl(D)⊆IFNCl(C). Thus IFNCl(D)⊆V and so D is IFN*g-closed.

Theorem 3.5

Every IFNCS is IFN*g-closed set.

Proof:

Let B⊆V and V is IFNgOS in $\tau_R'(X)$. Since B is IFNCS, IFNCl(B)⊆B. That is IFNCl(B)⊆B⊆V. Hence B is an IFN*g-closed set.

Theorem 3.6

An IFN*gCS A is IFNCS if and only if IFNCl(A) −A is IFNCS.

Proof:

Necessity:

Let A be IFNCS. Then IFNCl(A) = A and so IFNCl(A) −A=ϕ which is IFNCS.

Sufficiency:

Suppose IFNCl(A) −A is IFNCS. Then IFNCl(A) −A=ϕ since A is IFNCS. That is, IFNCl(A)=A or A is IFNCS. Therefore, A is IFN*g-closed set.

Theorem 3.7

Suppose that B⊆A⊆U, B is an IFN*g-closed set relative to A and that A is an IFN*g-closed subset of U. Then B is IFN*g-closed relative to U.

Proof:

Let B⊆V and suppose that V is IFNgOS in U. Then B⊆A∩V. Therefore IFNCl(B)⊆A∩V. It follows that A∩IFNCl(B)⊆A∩V and A⊆V∪IFNCl(B). Since A is IFN*g-closed in U, we have IFNCl(A) ⊆ V∪IFNCl(B). Therefore IFNCl(B)⊆IFNCl(A)⊆V∪IFNCl(B) and so IFNCl(B)⊆V. Then B is IFN*g-closed relative to V.

IV. INTUITIONISTIC FUZZY NANO*GENERALIZED CONTINUOUS MAPPINGS

In this section we have introduced intuitionistic fuzzy nano*generalized continuous mappings and investigated some of their properties.

Definition 4.1

A mapping f: (U, τ_R'(X)) → (V,τ_R''(Y)) is called an intuitionistic fuzzy nano*generalized continuous (IFN*g continuous) mapping if $f^{-1}(V)$ is an IFN*gCS in (U, τ_R'(X)) for every IFNCS V of (V,τ_R''(Y)).

Theorem 4.2

Let f: (U, τ_R'(X)) → (V,τ_R''(Y)) be a mapping and f^{-1}(A) be an IFRCS in X for every IFNCS A in Y. Then f is an IFN*g continuous mapping.

Proof:

Let A be an IFNCS in Y and f^{-1}(A) be an IFRCS in X. Since every IFRCS is IFNCS. Since every IFNCS is IFN*gCS. Then f^{-1}(A) is an IFN*gCS in X. Hence f is an IFN*g continuous mapping.

Theorem 4.3

Every IFN continuous mapping is IFN*g continuous mapping.

Proof:

Let f: (U, τ_R'(X)) → (V,τ_R''(Y)) be an IFN continuous mapping. Let V be an IFNCS in Y. Then f^{-1}(V) is an IFNCS in X. Since every IFNCS is an IFN*gCS, f^{-1}(V) is an IFN*gCS in X. Hence f is an IFN*g continuous mapping.

Theorem 4.4

Every IFSP continuous mapping is an IFN*g continuous mapping.

Proof:

Let f: (U, τ'_R(X)) → (V,τ''_R(Y)) be an IFSP continuous mapping. Let V be an IFNCS in Y. Then f^{-1}(V) is an IFSPCS in X. Since every IFSPCS is an IFN*gCS, f^{-1}(V) is an IFN*gCS in X. Hence f is an IFN*g continuous mapping.

Theorem 4.5

A mapping f: (U, τ'_R(X)) → (V,τ''_R(Y)) is an IFN*g continuous mapping if and only if the inverse image of each IFOS in Y is an IFN*gOS in X.

Proof:

Necessity:

Let A be an IFOS in Y. This implies A^c is IFNCS in Y. Then $f^{-1}(A^c)$ is an IFN*gCS in X, by hypothesis. Since $f^{-1}(A^c)$ = $[f^{-1}(A)]^c$, f^{-1}(A) is an IFN*gOS in X.

Sufficiency:

Let A be an IFNCS in Y. Then A^c is an IFOS in Y. By hypothesis $f^{-1}(A^c)$ is IFN*gOS in X. Since $f^{-1}(A^c)$ = $(f^{-1}(A))^c$, $(f^{-1}(A))^c$ is an IFN*gOS in X. Therefore $f^{-1}(A)$ is an IFN*gCS in X. Hence f is an IFN*g continuous mapping.

Definition 4.6

An IFNTS (U, τ'_R(X)) is said to be IFN*g$T_{1/2}$ space if every IFN*g closed set is IFN closed set.

Theorem 4.7

Let f: (U, τ'_R(X)) → (V,τ''_R(Y)) be an IFN*g continuous mapping, then f is an IFN* continuous mapping if X is an IFN*g$T_{1/2}$space.

Proof:

Let V be an IFNCS in Y. Then f^{-1}(V) is an IFN*gCS in X, by hypothesis. Since X is an IFN*g$T_{1/2}$ space, f^{-1}(V) is an IFNCS in X. Hence f is an IFN*g continuous mapping.

Theorem 4.8

Let f: (U, τ'_R(X)) → (V,τ''_R(Y)) be an IFN*g continuous mapping and g: (V,τ''_R(Y)) → (W,τ'''_R(Z)) is an IFN continuous mapping then g∘f: (U,τ'_R(X)) → (W,τ'''_R(Z)) is an IFN*g continuous mapping.

Proof:

Let V be an IFNCS in Z. Then g^{-1}(V) is an IFNCS in Y, by hypothesis. Since f is an IFN*g continuous mapping, $f^{-1}(g^{-1}(V))$ is an IFN*gCS in X. Hence g∘f is an IFN*g continuous mapping.

V. CONCLUSION

In this paper, we introduce intuitionistic fuzzy nano*generalized closed sets and intuitionistic fuzzy nano*generalized continuous mappings. We also examine their characteristics and provide a detailed analysis. Furthermore, we compare these sets with other existing intuitionistic fuzzy open and closed sets.

VI. REFERENCES

[1] K. Atanassov, Intuitionistic fuzzy sets, Fuzzy sets and systems, 1986, 87-96.

[2] Bhuvaneswari K, Mythili Gnanapriya K. Nano generalised closed sets in nano topological space. International Journal of Scientific and Research Publications. 2014; 4:5.

[3] D. Coker, An introduction to intuitionistic fuzzy topological space, Fuzzy sets and systems, 1997, 81-89.

[4] H. Gurcay, D. Coker, and Es. A. Haydar, On fuzzy continuity in intuitionistic fuzzy topological spaces, The J. fuzzy mathematics, 1997, 365-378.

[5] D. Jayanthi, Generalized β closed sets in intuitionistic fuzzy topological spaces, International Journal of Advance Foundation and research in Science & Engineering (IJAFRSE), December 2014.

[6] Lellis Thivagar M, Carmel Richard. On nano continuity. Mathematical Theory and Modelling. 2013;3(7):32-37.

[7] LellisThivagar M, Carmel Richard. On nano forms of weakly open sets. International Journal of Mathematics and Statistics Invention. 2013;1(1):31-37.

[8] Ramachandran M, Stephan Antony Raj A. Intuitionistic fuzzy nano topological space: Theory and Application (Accepted).

[9] M. Saranya, and D. Jayanthi, On Intuitionistic fuzzy β generalized closed sets, International Journal of Computational Engineering Research (IJCER), March 2016, 2250-3005.

[10] M. Saranya, and D. Jayanthi, On Intuitionistic fuzzy β generalized open sets, (Submitted).

[11] M. Saranya, and D. Jayanthi, On Intuitionistic fuzzy β generalized $T_{1/2}$ spaces, (Accepted).

[12] S. S Thakur, and Rekha Chaturvedi, Regular generalized closed sets in intuitionistic fuzzy topological spaces, Universitatea Din Bacau, Studii Si Cercetari Stiintifice, Seria: Mathematica, 2006, 257-272.

[13] L.A. Zadeh, Fuzzy sets, Information and control, 8 (1965) 338-353.