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LABELING ON WHEEL RELATED GRAPHS

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ABSTRACT

In this paper we investigate the existence of Tribonacci product cordial labelling on wheel related graphs.

Keywords: Wheel Graphs, Tribonacci Product Cordial Labeling.

I. INTRODUCTION

Bala et.al., [3] introduced the concept of Tribonacci product cordial labeling. An injective function $\delta : R(G) \rightarrow \{T_1, T_2, \dots T_m\}$ is said to be Tribonacci product cordial labelling if the induced function δ^*: $B(G) \rightarrow \{0,1\}$ defined by $\delta^*(r_i r_j) = (\delta(r_i)\delta(r_j))(mod2)$ satisfies the condition $| b_{\delta^*}(0) - b_{\delta^*}(1)| \leq 1$. A graph which admits Tribonacci Product cordial labelling is called Tribonacci product cordial graph. Motivated by this study, in this paper we investigate the existence of Tribonacci product cordial labelling on wheel related graphs.

II. PRELIMINARIES

In this section, we discuss the notion of a few wheel related graphs relevant to this work.

Friendship graph:

The **Friendship graph** F_m can be constructed by joining n copies of the cycle graph C_3 with a common vertex. F_m is a planar undirected graph with $2m + 1$ vertices and 3m edges.

Double wheel ;

A **Double wheel** graph is obtained from $CH_{1,m}$ by adding the vertices u_i; where $1 \leq i \leq m$ of $CH_{1,m}$ to its central vertex r_0, or in other words a double-wheel graph, denoted by $DW_{1,m}$ is the graph obtained by connecting the vertices of two (disjoint) cycles each of size m to a common vertex called central vertex. That is, $DW_{1,m} = 2C_m + K_1$. It has $2m + 1$ vertices and 4m edges.

Sunflower graph:

The **Sunflower graph** SF_m is the graph obtained by taking a wheel with the apex vertex r_0 and the consecutive rim vertices $r_1, r_2, r_3, \dots r_m$ and additional vertices $w_1, w_2, w_3, \dots w_m$ where w_i is joined by edges to r_i and r_{i+1}. It has $2m + 1$ vertices 4m edges.

III. MAIN RESULT

In this section we investigate the existence of Tribonacci Product Cordial Labeling on Friendship graph, Double wheel, Sunflower graph.

THEOREM : 3.1

Friendship graph F_m admits Tribonacci Product Cordial labeling

Proof

From the structure of Friendship graph F_m, it is clear that has $2m + 1$ vertices and $3m$ edges.

The vertex set and edge set are defined as follows:

$$R(G) = \{r_1, r_2, r_3, \dots, r_{2m+1}\}$$

$$E(G) = \{\{r_i r_{2m+1} / 1 \leq i \leq 2m\} \cup \{r_{2i-1} r_{2i} / 1 \leq i \leq m\}\}$$

Define the function $\delta: R \rightarrow \{\{T_1, T_2, T_3, , \dots . T_m\}\}$ to label the vertex as follows:

(i) $\delta(r_1) = T_{2m+1}$

(ii) $\delta(r_{2m+1}) = T_1$

For $2 \leq i \leq 2m$

(iii) $\delta(r_i) = T_i$

To obtain the edge labels, define the induced function $\delta^* : B \rightarrow \{0,1\}$ defined by $\delta^*(r_i r_j) = (\delta(r_i)\delta(r_j))(mod2)$.

Thus using the induced function the edges receive the labels as follows:

Case(i): $m \equiv 1(mod2)$

(i) $\delta^*(r_1 r_{2m+1}) = 0$
(ii) $\delta^*(r_2 r_{2m+1}) = 1$
(iii) $\delta^*(r_1 r_2) = 0$

For $1 \le i \le \frac{m-1}{2}$

(iv) $\delta^*(r_{4i-1} r_{2m+1}) = 0$
(v) $\delta^*(r_{4i} r_{2m+1}) = 0$
(vi) $\delta^*(r_{4i+1} r_{2m+1}) = 1$
(vii) $\delta^*(r_{4i+2} r_{2m+1}) = 1$
(viii) $\delta^*(r_{4i-1} r_{4i}) = 0$
(ix) $\delta^*(r_{4i+1} r_{4i+2}) = 0$

So, $|B_{\delta^*}(0) - B_{\delta^*}(1)| = \left|\frac{3m+1}{2} - \frac{3m-1}{2}\right| = 1$

Case(ii): $m \equiv 0 (mod 2)$

For $1 \le i \le \frac{m}{2}$

(i) $\delta^*(r_{4i-3} r_{2m+1}) = 1$
(ii) $\delta^*(r_{4i-2} r_{2m+1}) = 1$
(iii) $\delta^*(r_{4i-1} r_{2m+1}) = 0$
(iv) $\delta^*(r_{4i} r_{2m+1}) = 0$
(v) $\delta^*(r_{4i-3} r_{4i-2}) = 1$
(vi) $\delta^*(r_{4i-1} r_{4i}) = 0$

So, $|B_{\delta^*}(0) - B_{\delta^*}(1)| = \left|\frac{3m}{2} - \frac{3m}{2}\right| = 0$

Hence the condition $|B_{\delta^*}(0) - B_{\delta^*}(1)| \le 1$ is satisfied.

Therefore, the Friendship graph F_m admits Tribonacci product cordial labeling.

EXAMPLE : 3.1

Tribonacci Product Cordial labeling for Friendship graph F_m is shown below:

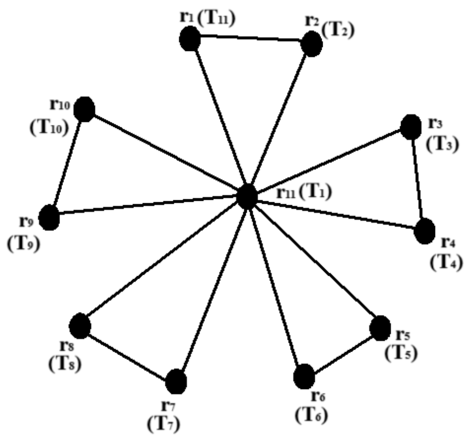

Figure 3.1.1

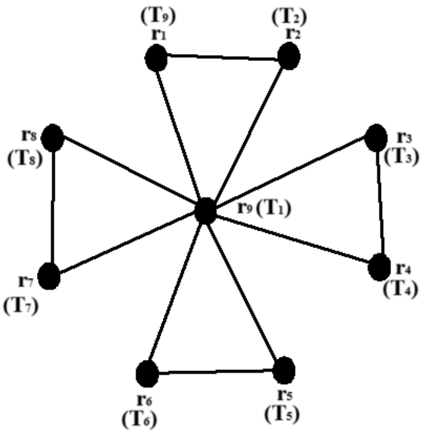

Figure 3.1.2

THEOREM : 3.2

Sun Flower graph SF_m admits Tribonacci Product Cordial labeling

Proof

From the structure of Sun Flower graph F_m, it is clear that has $2m + 1$ vertices and $4m$ edges.

The vertex set and edge set are defined as follows:

$$R(G) = \{\{r_1, r_2, r_3, \ldots, r_m\} \cup \{u\} \cup \{w_1, w_2, w_3, \ldots, w_m\}\}$$

$$E(G) = \{\{r_i r_{i+1} / 1 \le i \le m-1\} \cup \{r_i w_i / 1 \le i \le m\} \cup \{r_i u / 1 \le i \le m\} \cup \{r_{i+1} w_i / 1 \le i \le m-1\} \cup \{r_1 w_m\}\}$$

Define the function $\delta: R \to \{\{T_1, T_2, T_3, \ldots . T_m\}\}$ to label the vertex as follows:

$\boldsymbol{Case(i): m \equiv 1(mod2)}$

For $1 \le i \le \frac{m+1}{2}$

(i) $\delta\ (u) = T_1$
(ii) $\delta\ (r_{2i-1}) = T_{4i-2}$
(iii) $\delta\ (w_{2i-1}) = T_{4i-1}$

For $1 \le i \le \frac{m-1}{2}$

(i) $\delta\ (r_{2i}) = T_{4i+1}$
(ii) $\delta\ (w_{2i}) = T_{4i}$

$\boldsymbol{Case(ii): m \equiv 0(mod2)}$

For $1 \le i \le \frac{m}{2}$

(i) $\delta\ (u) = T_1$
(ii) $\delta\ (r_{2i-1}) = T_{4i-2}$
(iii) $\delta\ (w_{2i-1}) = T_{4i-1}$
(iv) $\delta\ (r_{2i}) = T_{4i+1}$
(v) $\delta\ (w_{2i}) = T_{4i}$

To obtain the edge labels, define the induced function $\delta^* : B \to \{0,1\}$ such that $\delta^*(r_i r_j) = (\delta(r_i)\delta(r_j))(mod2)$. Thus using the induced function the edges receive the labels as follows:

For $1 \le i \le m$

(i) $\delta^*\ (ur_i) = 1$
(ii) $\delta^*\ (r_i w_i) = 0$

For $1 \le i \le m-1$

(iii) $\delta^*\ (r_i r_{i+1}) = 1$
(iv) $\delta^*\ (r_{i+1} w_i) = 0$
(v) $\delta^*\ (r_1 r_m) = 1$
(vi) $\delta^*\ (r_1 w_m) = 0$

So, $|B_{\delta^*}(0) - B_{\delta^*}(1)| = |2m - 2m| = 0$

Hence the condition $|B_{\delta^*}(0) - B_{\delta^*}(1)| \le 1$ is satisfied.

Therefore, the Sun Flower graph SF_m admits Tribonacci product cordial labeling.

EXAMPLE : 3.2

Tribonacci Product Cordial labeling for Sun Flower graph SF_m is shown in the figure 3.2.1 and 3.2.2 respectively.

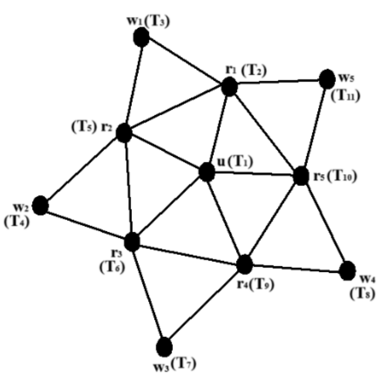

Figure 3.2.1

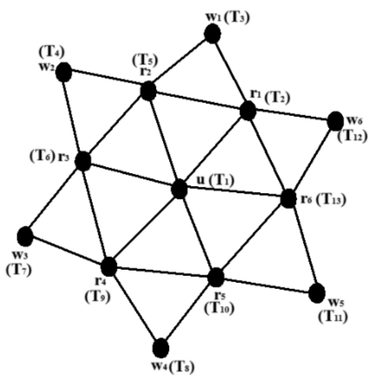

Figure 3.2.2

THEOREM : 3.3

Double Wheel graph $W_{m,m}$ admits Tribonacci Product Cordial labeling

Proof

From the structure of Double Wheel graph $W_{m,m}$, it is clear that has $2m + 1$ vertices and $4m$ edges.

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The vertex set and edge set are defined as follows:

$$R(G) = \{\{r_1, r_2, r_3, .., r_m\} \cup \{u\} \cup \{w_1, w_2, w_3, .., w_m\}\}$$

$$E(G) = \{\{ r_i r_{i+1}/1 \le i \le m-1 \} \cup \{ uw_i/1 \le i \le m\} \cup \{ r_i u/1 \le i \le m\} \cup \{ w_i w_{i+1}/1 \le i \le m-1 \}\}$$

Define the function $\delta: R \to \{\{T_1, T_2, T_3, , \ldots . T_n\}\}$ to label the vertex as follows:

$Case(i): m \equiv 1(mod2)$

For $1 \le i \le \frac{m+1}{2}$

(i) $\delta\ (u) = T_1$
(ii) $\delta\ (r_{2i-1}) = T_{4i-2}$
(iii) $\delta\ (w_{2i-1}) = T_{4i-1}$

For $1 \le i \le \frac{m-1}{2}$

(iv) $\delta\ (r_{2i}) = T_{4i+1}$
(v) $\delta\ (w_{2i}) = T_{4i}$

$Case(ii): m \equiv 0(mod2)$

For $1 \le i \le \frac{m}{2}$

(vi) $\delta\ (u) = T_1$
(vii) $\delta\ (r_{2i-1}) = T_{4i-2}$
(viii) $\delta\ (w_{2i-1}) = T_{4i-1}$
(ix) $\delta\ (r_{2i}) = T_{4i+1}$
(x) $\delta\ (w_{2i}) = T_{4i}$

To obtain the edge labels, define the induced function $\delta^* : B \to \{0,1\}$ given by $\delta^*(r_i r_j) = (\delta(r_i)\delta(r_j))(mod2)$. Thus using the induced function the edges receive the labels as follows:

For $1 \le i \le m$

(i) $\delta^*\ (ur_i) = 1$
(ii) $\delta^*\ (uw_i) = 0$

For $1 \le i \le m-1$

(iii) $\delta^*\ (r_i r_{i+1}) = 1$
(iv) $\delta^*\ (w_{i+1} w_i) = 0$
(v) $\delta^*\ (r_1 r_m) = 1$
(vi) $\delta^*\ (w_1 w_m) = 0$

So, $|B_{\delta^*}(0) - B_{\delta^*}(1)| = |2m - 2m| = 0$

Hence the condition $|B_{\delta^*}(0) - B_{\delta^*}(1)| \le 1$ is satisfied.

Therefore, the Double Wheel graph $W_{m,m}$ admits Tribonacci product cordial labeling.

EXAMPLE : 3.3

Tribonacci Product Cordial labeling for Double Wheel graph $W_{m,m}$ is shown in the figure 3.3.1 and 3.3.2 respectively.

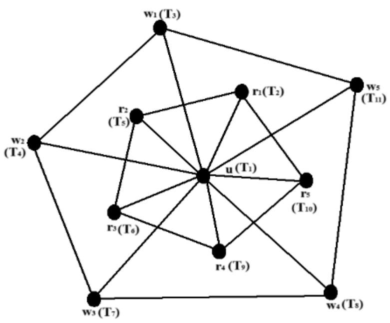

Figure 3.3.1

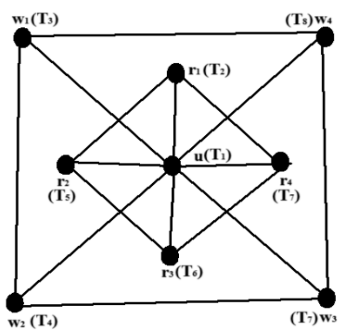

Figure 3.3.2

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IV. CONCLUSION

In this paper, we have confirmed the existence of Tribonacci Product Cordial Labeling on Friendship graph, Double wheel and Sunflower graph.

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