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SOME CONTRIBUTION TO INTRA-REGULAR Γ-SEMIHYPERRINGS

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ABSTRACT

The properties of intra-regular Γ-Semihyperring are studied. Also several characterizations of intra-regular Γ-Semihyperring with the help of ideals, bi-ideals and quasi-ideals of Γ-Semihyperrings has been done.

Keywords: T-Semihyperings, Bi-Ideals, Quasi-Ideals, Regular Γ-Semihyperings, Idempotent Set.

I. INTRODUCTION

In mathematics everyone is familiar to the classical algebraic structure in which composition of two elements is an element. But there is another view of mathematics in hyperstructure theory in which the composition of two element becomes a set. In 1934, the notion of hyperstructure was first introduced by French mathematician Marty when he presented a paper in conference. But still the theory of hyperstructure was not popular amongst the mathematicians in the word. After time passes it is found that the theory of hyperstructure has vast application in various branches of science and then theory of hyperstructure becomes popular and being studied by mathematicians across the word. In 2003, Corsini and Leoreanu [1] have given application of theory of hyperstructures in various subjects like: geometry, cryptography, artificial intelligence, relation algebras, automata, median algebras, relation algebras, fuzzy sets and codes. If we let H be a non-empty set. Then, the map $o : H \times H \longrightarrow P^*(H)$ is called a hyperopertion, where $P^*(H)$ is the family of all non-empty subsets of H and the couple (H, o) is called a hypergroupoid. Moreover, the couple (H, o) is called a semihypergroup if for every $a, b, c \in H$ we have, $(aob)o\ c = ao(boc)$.

As theory of hyperstructure has vast application in various fields of sciences so it is essential to study the concepts of classical algebraic structure in hyperstructure theory. The main aim of this paper is to study the concepts of classical algebraic structure to a hyperstructure theory. In [3], Jagatap and Pawar introduced intra-regular Γ-Semiring and made its characterizations with the help of ideals Γ-Semirings. Here we have introduced the concept of intra-regular Γ-Semihyperring and made its characterizations with the help of ideals, bi-ideals and quasi-ideals of Γ-Semihyperrings analogues to Jagatap and Pawar [3].

II. PRELIMINARIES

Here are some useful definitions and the readers are requested to refer [9].

Definition 1.1. [9] Let R be a commutative semihypergroup and Γ be a commutative group. Then, R is called a Γ-semihyperring if there is a map $R \times \Gamma \times R \longrightarrow P^*(H)$ (images to be denoted by $a\alpha b$, for all $a, b \in R$ and $\in \Gamma$) and $P^*(R)$ is the set of all non-empty subsets of R satisfying the following conditions:

(1) $a\alpha(b + c) = a\alpha b + a\alpha c$.

(2) $(a + b)\alpha c = a\alpha c + b\alpha c$.

(3) $a(\alpha + \beta)c = a\alpha c + a\beta c$.

(4) $a\alpha(b\beta c) = (a\alpha b)\beta c$, for all $a, b, c \in R$ and for all $\alpha, \beta \in \Gamma$.

Definition 1.2. [9] A Γ-semihyperring R is said to be commutative if $a\alpha b = b\alpha a$ for all $a, b \in R$ and $\alpha \in \Gamma$.

Definition 1.3. [9] A Γ-semihyperring R is said to be with zero, if there exists $0 \in R$ such that $a \in a + 0$ and $0 \in 0\alpha a$, $0 \in a\alpha 0$ for all $a \in R$ and $\alpha \in \Gamma$.

Let A and B be two non-empty subsets of a Γ-semihyperring R and $x \in R$, then

$$A + B = \{x | x \in a + b, a \in A, b \in B\}$$

$$A\Gamma B = \{x\ | x \in a\alpha b, a \in A, b \in B, \alpha \in \Gamma\}$$

Definition 1.4. [9] A non-empty subset R_1 of Γ-semihyperring R is called a Γ -subsemihyperring if it is closed with respect to the multiplication and addition, that is, $R_1 + R_1 \subseteq R_1$ and $R_1\Gamma R_1 \subseteq R_1$.

Definition 1.5. [9] A right (left) ideal I of a Γ-semihyperring R is an additive sub semihypergroup of $(R,+)$ such that $I\Gamma R \subseteq I$ $(R\Gamma I \subseteq I)$. If I is both right and left ideal of R, then we say that I is a two sided ideal or simply an ideal of R.

Definition 1.6. [8] A non-empty set B of Γ-semihyperring R is a bi-ideal of R if B is a Γ-subsemihyperring of R and $B\Gamma R\Gamma B \subseteq B$.

Definition 1.7. [5] A subsemihyper group Q of $(R,+)$ is said to be a quasi-ideal of Γ -semihyperring R if $(R\Gamma Q) \cap (Q\Gamma R) \subseteq Q$.

Definition 1.8. [4] An element e of Γ-semihyperring R is said to be a left (right) identity of R if $r \in e\alpha r$ $(r \in r\alpha e)$ for all, $r \in R$ and $\alpha \in \Gamma$.

An element e of Γ-semihyperring R is said to be a two sided identity or simply an identity if e is both left and right identity, that is $r \in e\alpha r \cap r\alpha e)$ for all $r \in R$ and $\alpha \in \Gamma$

Theorem 1.9. [8] Every quasi-ideal of a Γ-Semihyperring R is a bi-ideal of R.

Theorem 1.10. [8] Every one sided (two sided) ideal of a Γ-Semihyperring R is a bi-ideal of R.

Theorem 1.11. [5] Every one sided (two sided) ideal of a Γ-Semihyperring R is a quasi-ideal of R.

Definition 1.12. [6] An ideal generated by subset A of a Γ-Semihyperring R is the smallest ideal of R containing A. It is denoted as <A>.

Theorem 1.13. [6] Let R be a Γ-Semihyperring. Then for subset A of R,$< A >= A \cup A\Gamma R \cup R\Gamma A \cup R\Gamma A\Gamma R$.

Definition 1.14. [4] A subset E of a Γ-Semihyperring R is said to be a idempotent subset of R if there exists $\Gamma_1 \subseteq \Gamma$ such that $E\Gamma_1 E \subseteq E$.

III. INTRA-REGULAR Γ-SEMIHYPERRINGS

In this section, we introduced the concept of intra-regular Γ-Semihyperring and studied some basic properties in this respects on the line of Jagatap and Pawar [3]. Throughout this paper we consider that Γ-Semihyperring has identity element.

Definition 2.1. A Γ-Semihyperring S is said to be an intra-regular Γ-Semihyperring if for any $x \in S$, $x \in S\Gamma x\Gamma x\Gamma S$.

Theorem 2.2. Let S be an intra-regular Γ-Semihyperring. Then ideal of S is an idempotent ideal.

Proof. Let S be an intra-regular Γ-Semihyperring and I be an ideal of S. For any $a \in I$, we have $a \in S\Gamma a\Gamma a\Gamma S$ as S is an intra-regular. Therefore $a \in S\Gamma a\Gamma a\Gamma S \subseteq S\Gamma I\Gamma I\Gamma S$ as $a \in I$. Hence $a \in S\Gamma a\Gamma a\Gamma S \subseteq (S\Gamma I)\Gamma(I\Gamma S) \subseteq I\Gamma I$. Thus $a \in I$ gives that $a \in I\Gamma I$. Hence we get $I \subseteq I\Gamma I$. Also, $I\Gamma I \subseteq I$ holds always. Therefore we get $I = I\Gamma I$, that is any ideal of an ideal of S is an idempotent ideal.

Theorem 2.3. Let I be an ideal of an ideal of a Γ-Semihyperring S. Then $< I >^3 = < I > \Gamma < I > \Gamma < I > \subseteq I$.

Proof. Let I be an ideal of an ideal K of a Γ-Semihyperring S. Then we have,

$$< I > \Gamma < I > \Gamma < I > \subseteq K\Gamma < I > \Gamma K \quad (\text{Since } < I > \subseteq K)$$
$$= K\Gamma(I \cup I\Gamma S \cup S\Gamma I \cup S\Gamma I\Gamma S)\Gamma K \quad (\text{By theorem 1.13})$$
$$= (K\Gamma I \cup K\Gamma I\Gamma S \cup K\Gamma S\Gamma I \cup K\Gamma S\Gamma I\Gamma S)\Gamma K$$

As I is an ideal of K, we have $K\Gamma I \subseteq I, I\Gamma K \subseteq I$. Also we have, $K\Gamma S \subseteq K, S\Gamma K \subseteq K$, since K is an ideal of S. Therefore,

$$< I > \Gamma < I > \Gamma < I > \subseteq (I \cup I\Gamma S \cup K\Gamma I \cup K\Gamma I\Gamma S)\Gamma K$$
$$\subseteq (I \cup I\Gamma S \cup I \cup I\Gamma S)\Gamma K$$
$$= I\Gamma K \cup I\Gamma S\Gamma K \cup I\Gamma K \cup I\Gamma S\Gamma K$$
$$\subseteq I \cup I\Gamma K \cup I \cup I\Gamma K$$
$$\subseteq I \cup I \cup I \cup I = I.$$

Hence the theorem.

Theorem 2.4. In an intra-regular Γ-Semihyperring S, any ideal of an ideal of S is an ideal of S.

Proof. Let S be an intra-regular Γ-Semihyperring, K be an ideal of S and I be an ideal of K. By Theorem 2.2. any ideal of S is an idempotent ideal. Therefore $< I >^3 = < I > \Gamma < I > \Gamma < I \geq = < I > \Gamma < I > = < I >$. Hence by

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Theorem2.3. $<I>^3=<I>\Gamma<I>\Gamma<I>=<I>\subseteq I$. Thus we get, $<I>\subseteq I$. As $I\subseteq<I>$ always holds, we get $<I>=I$. This shows that I is an ideal of S.

For detail of semiprime, prime ideals of Γ-Semihyperring and Uniformly Strongly Prime Γ- semihyperrings for one can see [6,7].

Theorem 2.5. If S is an intra-regular Γ-Semihyperring, then any proper ideal of S is a semiprime ideal of S.

Proof. Let S be an intra-regular Γ-Semihyperring and P be a proper ideal of S. Let A be any ideal of S such that $A\Gamma A\subseteq P$. For any $a\in A$, we have $a\in S\Gamma a\Gamma a\Gamma S$ as S is intra-regular. Hence we have $S\Gamma a\Gamma a\Gamma S\subseteq S\Gamma A\Gamma A\Gamma S=(S\Gamma A)\Gamma(A\Gamma S)\subseteq A\Gamma A\subseteq P$, since A is an ideal of S and $A\Gamma A\subseteq P$. Therefore $a\in S\Gamma a\Gamma a\Gamma S\subseteq P$. Thus $a\in A$ implies $a\in P$. This shows that $A\subseteq P$. Therefore P is a semiprime ideal of S.

Theorem 2.6. Let S be a Γ-Semihyperring. Then S is intra-regular if and only if each right ideal R left ideal L of S satisfies $R\cap L\subseteq L\Gamma R$.

For a Γ-Semihyperring S one may define the interior ideal I of a Γ-Semihyperring as I is Γ-subsemihyperring such that $S\Gamma I\Gamma S\subseteq I$. Clearly we can see any ideal of S is a interior ideal of S.

Theorem 2.7. In Γ-Semihyperring S following statements are equivalent.

1. S is an intra-regular.
2. For a bi-ideal B, a right ideal R and an interior ideal K of$S$, $K\cap B\cap R\subseteq K\Gamma B\Gamma R$.
3. For a bi-ideal B, a right ideal R and an ideal I of S, $I\cap B\cap R\subseteq I\Gamma B\Gamma R$.
4. For a Quasi-ideal Q, a right ideal R and an interior ideal K of S, $K\cap Q\cap R\subseteq K\Gamma Q\Gamma R$.
5. For a Quasi-ideal Q, a right ideal R and an interior ideal I of S, $I\cap Q\cap R\subseteq I\Gamma Q\Gamma R$.

Proof. (1) ⇒ (2)

Assume S is intra-regular. Let R be a right ideal, K be an interior ideal and B be a bi-ideal of S with $a\in K\cap B\cap R$. But as S is an intra-regular we have,

$$\begin{aligned} a\in S\Gamma a\Gamma a\Gamma S &\subseteq (S\Gamma a)\Gamma(S\Gamma a\Gamma a\Gamma S)\Gamma S \\ &= (S\Gamma a\Gamma S)\Gamma a\Gamma(a\Gamma S\Gamma S) \\ &\subseteq (S\Gamma K\Gamma S)\Gamma B\Gamma(R\Gamma S\Gamma S) \\ &\subseteq K\Gamma B\Gamma R \quad \text{(Since } K \text{ is an interior ideal and } R \text{ is a right ideal)} \end{aligned}$$

Hence we get, $S\Gamma a\Gamma a\Gamma S\subseteq K\Gamma B\Gamma R$. Therefore we get, $a\in K\cap B\cap R$ implies that $a\in K\Gamma B\Gamma R$. This shows that $K\cap B\cap R\subseteq K\Gamma B\Gamma R$.

(2) ⇒ (3) , (4) ⇒ (5) Implication follows easily since every ideal is an interior ideal.

(3) ⇒ (5), (2) ⇒ (4) Implication follows easily since every quasi-ideal is a bi-ideal.

(5) ⇒ (1)

Let L be a left ideal and R be a right ideal of S. We know a left ideal L is a quasi-ideal of S. Therefore by (5), we have, $S\cap L\cap R\subseteq S\Gamma L\Gamma R$. Hence $L\cap R\subseteq(S\Gamma L\Gamma R)\subseteq L\Gamma R$. By Theorem 2.6. we get, S is an intra-regular Γ-Semihyperring.

Theorem 2.8. In Γ-Semihyperring S following statements are equivalent.

1. S is an intra-regular.
2. For a bi-ideal B, a left ideal L and an interior ideal K of S, $K\cap B\cap L\subseteq L\Gamma B\Gamma K$.
3. For a bi-ideal B, a left ideal L and an ideal I of S, $I\cap B\cap L\subseteq L\Gamma B\Gamma I$.
4. For a quasi-ideal Q, a left ideal L and an interior ideal K of S, $K\cap Q\cap L\subseteq L\Gamma Q\Gamma K$.
5. For a quasi-ideal Q, a left ideal L and an ideal I of S, $I\cap Q\cap L\subseteq L\Gamma Q\Gamma I$.

Proof. (1) ⇒ (2) Asuume S is an intra-regular. Let L be a left ideal, K be an interior ideal and B be a bi-ideal of S. For any $a\in K\cap B\cap L$, we have

$$\begin{aligned} a\in S\Gamma a\Gamma a\Gamma S &\subseteq S\Gamma(S\Gamma a\Gamma a\Gamma S)\Gamma a\Gamma S \\ &= (S\Gamma S\Gamma a)\Gamma a\Gamma(S\Gamma a\Gamma S) \\ &\subseteq (S\Gamma S\Gamma L)\Gamma B\Gamma(S\Gamma K\Gamma S) \\ &\subseteq L\Gamma B\Gamma K \quad \text{(Since } L \text{ is a left ideal and } K \text{ is an interior ideal)} \end{aligned}$$

Therefore $a\in K\cap B\cap L$ implies $a\in L\Gamma B\Gamma K$. Thus we get, $K\cap B\cap L\subseteq L\Gamma B\Gamma K$.

(2) ⇒ (3) , (4) ⇒ (5) Implication follows easily since every ideal is an interior ideal.

(3) ⇒ (5), (2) ⇒ (4) Implication follows easily since every quasi-ideal is a bi-ideal.

(5) ⇒ (1)

Let L be a left ideal and R be a right ideal of S. We know a right ideal R is a quasi-ideal of S. Therefore by (5), we have, $S \cap L \cap R \subseteq S\Gamma L\Gamma R$. Hence $L \cap R \subseteq (S\Gamma L\Gamma R) \subseteq L\Gamma R$. By Theorem 2.6. we get, S is an intra-regular Γ-Semihyperring.

IV. RESULTS AND DISCUSSION

We found that all the results of intra-regular Γ-Semiring almost holds in Γ-Semihyperring. Also we found that the result "Let S be a Γ-Semihyperring. Then S is intra-regular if and only if each right ideal R left ideal L of S satisfies $R \cap L \subseteq L\Gamma R$" is very important amongst the results we have proved.

V. CONCLUSION

The conclusion has been made that there is lot of scope for to study different concepts of classical algebraic structure to a hyperstructure theory. Also in our preset paper there is lot of scope to study the results with various examples. If we consider that the Γ-Semihyperring S is regular as well as intra-regular it will be characterized with the help different types of ideals of Γ-Semihyperring.

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