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FRACTIONAL INTEGRAL OPERATORS ASSOCIATED WITH LAPLACE TRANSFORM

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ABSTRACT

In the present paper, we discussed the application of Fractional integral operators Associated with Laplace transform. convolution of I-Function to solve homogenous and non-homogenous linear fractional differential equations using Riemann-Liouville differential operator. Fractional calculus, Laplace transform is a generalization of the Laplace transform both classical sense of the definition as in their properties simplicity, efficiency and the high accuracy.

Fractional Laplace transforms are powerful and efficient techniques for obtaining analytic solution of homogenous and non-homogenous linear fractional differential equations and are in uniformity with the solutions available in the literature.

AMS 2020 Subject classification No. 26A33, 44A10, 45E10 and theorems. In this work we study the response of the α-Integral Laplace transform introduced on the fractional derivative of Riemann-Liouville. Shehu transform is a generalization othe Laplace integral transform for solving fractional differential equations in the time domain and is applied to both ordinary and fractional differential equations to show its.

Keywords: Fractional Integral Operators, Convolution, I-Funcion, Laplace Transform.

I. INTRODUCTION

Fractional Calculus is a field of mathematical study that grows out of the classical calculus and being a generalization of classical calculus, it preserves many of its fundamental properties .The fractional calculus plays crucial role in many fields of pure and applied mathematics. One of the most effective methods to solve differential equations is to use integral transforms. Literally, the origination of the integral transforms can be traced back to the work of P.S. Laplace in1780s and Joseph Fourier in1822.Fractional integrals and derivatives, in association with different integral transforms, are widely used to solve different types of differential and integral equations. Fractional differential equations are extensively used in interpretation and modeling in applied mathematics and physics including fluid flow, rheology, electrical circuits, probability and statistics, control theory of dynamical system, viscoelasticity, chemical physics, optics and signal processing and so on[7]. The gist of this paper is to present Fractional Laplace Transform and Our purpose is to extend the application of these methods to obtain the exact solution of homogenous and non-homogenous linear fractional differential equations. The paper is organized as follows. Some necessary definitions and preliminaries of fractional calculus theory are introduced in the Laplace Transform . the object of this paper is to solve an integral equation of convolution form having H- function of two variable as it's kernel. Some known results are obtained as special cases beginning. In section 2, we present properties and Lemma's related to α-Integral Laplace A new class of convolution integral equations whose kernels involve an H-Function of several variables, which is defined by a multiple contour integral of the Mellin-Barnes type, is solved. It is also indicated how the main theorem can be specialized to derive a number of (known or new) results on convolution integral equations involving simpler special functions of interest in problems of applied mathematics and mathematical physics.

In the present paper a convolution integral equation of Fredholm type whose kernel involves a product of generalized polynomial set, general multivariable polynomials, Fox's H-function and H -function, has been solved by using the theory of Mellin transforms. Our main result is believed to be general and unified in nature. A number of (known and new) results follow as special cases by specializing the coefficients and parameters involved in the kernel.

The following definition and results will be required in this paper

(I)The Laplace Transform if

$$F(P) = L[f(t);p] = \int_0^\infty e^{-pt} f(t)dt \quad , \quad \mathrm{Re}(p) > 0 \qquad ...(1.1)$$

Then F(p) is called the Laplace transform of f(t) with parameter p and is represented by F(p) = f (t) Erdelyi [(3) pp.129-131]

$$L[f(t);p] = F(P) \text{ then } L[e^{-at} f(t)] = F(p+a) \qquad ...(1.2)$$

And if $f(0) = f'(0) = f''(0) = ... = f^{m-1}(0) = 0 \quad , \; f^n(t)$

Is continuous and differential, then

$$L[f^n(t);p] = P^n F(p) \qquad ....(1.3)$$

If $L[f_1(t)] = F_1(p)$ then $L[f_2(t)] = F_2(p)$

Then convolution theorem for Laplace transform is

$$L\{\int_0^1 f_1(t) f_2(t-u)du\} = L\{f_1(t)\} L\{f_2(t)\} = F_1(p).F_2(p) \qquad ...(1.4)$$

The H-Function Defined by Saxena and kumbhat [1] is an extension of Fox's H-Function on specializing the parameters , H-Function can be reduced to almost all the known special function as well as unknown

(II) Let α, β and η be complex numbers,and let x ∈ R_+= (0, ∞) Follwing Saigo [8} Fractional integral (Re(α)>0 and derivative Re(α) < 0 of first kind of a function f(x) on R_+ are defined respectively in the forms:

$$I_{0,\,x}^{\alpha,\beta,\eta}(f) = \frac{x^{-\alpha-\beta}}{\Gamma(\alpha)} \int_0^x (x-t)^{\alpha-1}\, 2F1\left(\alpha+\beta\,,\, -\eta\,;\, \alpha\,;\, 1-\frac{t}{x}\right) f(t)dt\,; \qquad ...(1.5) \qquad \mathrm{Re}(\alpha)>0$$

$$\frac{d^n}{dt^n} I_{0,\,x}^{\alpha+n\;\beta-n\;,\eta-n}(f)\,,\; 0< \mathrm{Re}(\alpha)+n<1 \qquad (n= 1, 2, 3, ...),$$

Where 2F1(a, b; c; z) is Gauss's hypergeometric function

Fractional integral Re(α)>0 and derivative Re(α) < 0 of second kind a function f(x) on R_+ are given by:

$$J_{x,\infty}^{\alpha,\beta,\eta} = \frac{1}{\Gamma(\alpha)} \int_0^x (x-t)^{\alpha-1}\, t^{\alpha-\beta}\, {}_2F_1\left(\alpha+\beta\,,\, -\eta\,;\, \alpha\,;\, 1-\frac{t}{x}\right) f(t)dt\,; \qquad ....(1.6) \qquad \mathrm{Re}(\alpha)>0$$

$$= (-1)^n \frac{d^n}{dt^n} j_{x,\,\infty}^{\alpha+n\;\beta-n\;,\eta}(f)\,, \qquad 0< \mathrm{Re}(\alpha)+n<1 \quad (n= 1, 2, 3, ...),$$

Let α, β,η and λ be complex numbers. Then there hold the following formulae. . the R.H.S. has a definite meaning

$$I_{0,\,x}^{\alpha,\beta,\eta}\; t^{\lambda} = \frac{\Gamma(1+\lambda)\Gamma(1+\lambda-\beta+\eta)}{\Gamma(1+\lambda-\beta)\Gamma(1+\lambda+\alpha+\eta)} x^{1-\beta} \qquad .....(1.7)$$

provided that Re(α)> max [0, Re(β-η)] — 1

(III) The I-Function which was recently introduced by saxena [9] is an extension of fox'S H-function. on Specializing the parameters,I-Function can bereduced to almost all the known special function as wellas unknowns. The I-Function of one variable is further studied by so many researchers notably as vaishya, jain, and verma [14], Sharma and shrivastava [12] , Sharma and Tiwari [13] ,Nair [11] with certain properties series summation, integration etc.

Saxena [10] represent and define the I-Function of one variable as follows

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$$I\,[Z] = I^{m,n}_{p_i;\,q_i;r}\left[Z\left|\begin{matrix}(a_j,\alpha_j)_{1,n}\ldots\ldots(a_{ji},\alpha_{ji})_{n+1,p_i}\\ \vdots\\ (b_j,\ \beta_j)_{1,m}\ldots\ldots(b_{ji},\beta_{ji})_{m+1,q_i}\end{matrix}\right.\right] \quad \ldots\ldots(1.8)$$

$$= \frac{1}{2\pi i}\int_L t(s)\ z^s\ ds.$$

Where

$$t(s) = \frac{\prod_{J=1}^{m}[b_j - \beta_j s\ \ \prod_{j+1}^{n}[1-a_j, +\alpha_j s}{\sum_{J=1}\left[\prod_{J=m+1}^{q_i}[1-bji+\beta jis\ \ \prod_{J=n+1}^{p_i}[a_{ji} - \alpha_{ji}s\right]} \quad \ldots\ldots(1.9)$$

The following definition and results will be required in this paper

II. MAIN RESULT

Result I

$$L\left\{e^{-nt}\ \ t^h\ I^{\alpha,\beta,\eta}_{x,\infty}\{t^\lambda\ I^{m,n}_{pi,qi,r}\left[zx^\lambda\left|\begin{matrix}(aj,\alpha j)\ldots\ldots(aji,\alpha ji)_{pi}\\ \vdots\\ (bj,\beta j)\ldots\ldots(bji,\beta ji)_{qi}\end{matrix}\right.\right],P\right\}$$

$$=\frac{x^{\lambda-\beta}\ \lceil 1+h}{(P+n)^{1+h}}\ \ I^{m\ \ n+2}_{p;+2\ \ q;+2;r}\left[zx^k\left|\begin{matrix}(-\lambda,k)\ (-\lambda-\eta+\beta,k)\ (aj,\alpha j)\ldots\ldots(aji,\alpha ji)_{pi}\\ \vdots\\ (-\lambda+\beta,k)(-\lambda-\alpha-\eta,k)\ (bj,\beta j)\ldots\ldots(bji,\beta ji)_{qi}\end{matrix}\right.\right] \quad \ldots.(2.1)$$

Provided (in addition to the appropriate convergence and existence conditions)that

λ, μ >0   Re(1+∝) > 0 Re(β) > 0 and Re(p) >0 1≥λ>a

Result II

$$L.h.s.\ \Rightarrow L\left\{e^{-nt}\ \ t^h\ I^{m,n}_{p_i.q_i;r}\left[zt^k\left|\begin{matrix}(a_j\ \alpha_j)\ldots\ldots(a_{ji}\ \alpha_{ji})p_i\\ \vdots\\ (b_j\ \beta_j)\ldots\ldots(b_{ji},\beta_{ji})_{qi}\end{matrix}\right.\right];P\right\}$$

$$= (p+n)^{-1-h}\ I^{m,n+1}_{p;+1\ q;:r}\left[z(p+n)^{-k}\left|\begin{matrix}(-h,k)(a_j\ \alpha_j)\ldots\ldots(a_{ji}\ \alpha_{ji})p_i\\ \vdots\\ (b_j\ \beta_j)\ldots\ldots(b_{ji},\beta_{ji})_{qi}\end{matrix}\right.\right] \quad \ldots\ldots(2.2)$$

Provided (in addition to the appropriate convergence and existence conditions)that

λ, μ >0   Re(1+∝) > 0 Re(β) > 0 and Re(p) >0 1≥λ>a

Proof I First Taking Fractional integral formula for one variable I- function. then apply the formula (1.12) for I-function. then using convolution of laplace transform for I-function.and then we get required result

Proof II First Taking Fractional integral formula for one variable I- function. then using convolution of laplace transform for I-function.and then we get required result

III. CONCLUSION

From this Paper we get some many solution of Fractional Differential operators Associated with Laplace transform of fractional integral.

The exact solutions of fractional differential calculus play a crucial role in mathematical physics. but it requires an observation of the term forcing, so not every fractional differential calculus with a constant coefficient can be solved by the method of Laplace transform. We apply the Laplace transform to fractional integral & convolution of I -Function. Analytical solutions of these models for various fractional orders and the solution of the corresponding classical equation were recovered as a particular case. We observe, in the various graphs studied, that the different values of the fractional-order of the derivative allow very different behaviors of the solution, especially in the time of convergence to the equilibrium state, which makes the model convenient to model, among others, growth phenomena.

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