

## INVESTIGATION ON THE INFLUENCE OF ELECTRO THERMO HYDRODYNAMICS (ETHD) ON SEMI-CIRCULAR CYLINDER

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### ABSTRACT

Electro-thermo-convective phenomena can be used to actively enhance heat transfer in dielectric liquids. The lattice Boltzmann method (LBM) is a well-established tool for simulating heat and fluid flows. The study focuses on the numerical investigation of electro-thermo-convection in a system consisting of a square enclosure and a heated inner semi-circular cylinder. Electro-Thermo convective heat transfer phenomena have wide range applications like heat transfer in electronic devices, heat exchangers, micro fluids, and so on. The flow movement is driven by thermal buoyancy and an electric field generated by injecting charges from the cylinder. A unified lattice Boltzmann method is employed to solve the governing equations, including the equations for Navier-Stokes equations, energy conservation, and simplified Maxwell's equations. The physical model being studied consists of a square cavity with a semi-circular cylinder at its center. The inner semi-circular cylinder has a higher electric potential and temperature, while the outer square cavity is grounded and has a lower temperature. The electrochemical reaction at the electrode-liquid interface generates free charges that are injected into the liquid. When the electrical effect is added, it boosts heat transfer, especially when the electric driving power (represented by  $T$ ) goes beyond a certain important level. When  $T$  is really high, the flow is mainly controlled by the Coulomb force, and the average Nusselt number value is also high indicating convection heat transfer takes place and properties of the fluid and the shape of the system affect this enhanced heat transfer.

**Keywords:** Electric Rayleigh Number, Nusselt Number.

### I. INTRODUCTION

Electro-Thermo-Hydro Dynamics is the study of how electric and magnetic fields influence fluid flow and heat transfer. It explores the dynamic behaviour of fluids in the presence of **electromagnetic forces** and temperature gradients. This field finds applications in diverse areas, including microfluidics, energy conversion, environmental science, and materials processing.

Fluid movement in ring-shaped spaces is important for cooling electronics, nuclear reactors, and heat exchangers. To make heat transfer better in these tight spaces. Electricity is used to make the fluid move, which helps with transferring heat and material more efficiently. This method has advantages like easy design, quick control, low energy use, and less noise

#### 1.1 Electromagnetic forces:

Electromagnetic forces play a fundamental role in shaping the behaviour of fluids when subjected to electric and magnetic fields. This phenomenon, known as Electromagnetic Forces in Fluids (EMFF), has wide-ranging applications across various scientific and technological domains. It exploration of how these forces influence fluid behaviour, inducing motion and altering the characteristics of the fluid.

Electro-Thermo-Hydrodynamics (ETHD) is a wide range field that studies the coupled interactions between electrical, thermal, and fluid dynamics phenomena. This emerging field is of increasing importance due to the growing number of applications that involve these three phenomena. **Electrical Effects, Thermal Effects, Fluid Dynamics Effects** For example, ETHD is essential for understanding the cooling of electronic devices, the design of energy conversion systems, and the operation of microfluidic devices. ETHD investigates the complex interactions between electric fields, temperature variations, and fluid motion within various systems. These

interactions give rise to complex behaviours. By studying the combined effects of these three phenomena, ETHD seeks to unravel the underlying mechanisms that govern their mutual influence.

### 1.2 Electrical Effects

The electrical effects in ETHD are typically due to the presence of an electric field. Electric fields can cause a number of phenomena, including:

**Electrostatic forces:** These forces can cause charged particles to move, which can lead to fluid flow.

$$F = k \cdot (q_1 q_2) / r^2$$

Where:

F is the electrostatic force between the two particles, in Newtons

K is a constant called the Coulomb constant, with a value of  $8.99 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$

q1 and q2 are the charges of the two particles, in Coulombs

R is the distance between the two particles, in meters

**Joule heating:** This occurs when an electric current flows through a conductor, and it can cause the conductor to heat up.

$$Q = I^2 R \cdot t$$

### Electric Fields in ETHD

Electric fields are areas in space where charged particles feel forces. In ETHD, these fields are important because they affect how charged particles and ions move within liquids or gases. When you have a fluid, like water or air, and you want to make it move or change its behaviour. You can do this by creating an electric field.

This electric field can be made using different methods. One way is by putting something like metal plates in the fluid and applying a voltage to them. When you do this, you're basically creating an invisible force that pushes and pulls on the charged particles in the fluid.

The electric field can make the fluid move. The field can make the particles in the fluid go in certain directions, creating flows and currents. This is really useful in things like tiny devices where you need to move liquids around, such as in labs or microfluidic chips.

The electric fields can change how charged particles behave in the fluid. They might stick to surfaces or move faster because of the field. This can be handy for things like separating different particles in a fluid or controlling chemical reactions.

So, in simple terms, electric fields in ETHD are like invisible hands that push and pull on charged particles in fluids, making them move around and behave in unique ways. They are like the remote control for liquids and gases, helping us do all sorts of cool things in science and technology!

**Electromigration:** This is the movement of atoms or ions under the influence of an electric field, and it can cause damage to materials.

### 1.3 Thermal Effects

The thermal effects in ETHD are typically due to the presence of a temperature gradient. Temperature gradients can cause a number of phenomena, including:

**Buoyancy forces:** These forces are caused by the difference in density between hot and cold fluids, and they can lead to fluid flow.

Buoyancy forces arise due to variations in fluid density, often caused by temperature differences. When a hotter fluid is less dense than its surroundings, it rises, displacing the cooler, denser fluid. This phenomenon, known as convection, can lead to fluid flow as the cycle repeats. This process is crucial in natural phenomena like ocean currents and atmospheric circulation. It's also a key factor in engineering applications such as heat transfer and fluid dynamics in various systems.

**Thermal expansion:** This is the expansion of materials due to an increase in temperature, and it can cause changes in the shape and size of objects.

**Convection:** This is the transfer of heat by the movement of fluids, and it is a major mechanism for heat transfer in many applications.

### 1.4 Fluid Dynamics Effects

Fluid dynamics is all about how fluids, like liquids and gases, move and interact with their surroundings. When fluid is in motion, it can create various effects that influence the behaviour of objects and the way heat and energy are transferred

**Drag forces:** These forces oppose the motion of a fluid, and they can be caused by friction between the fluid and a surface.

**Lift forces:** These forces act perpendicular to the direction of fluid flow, and they can be used to control the motion of objects.

**Turbulence:** This is a chaotic form of fluid flow, and it can cause mixing and heat transfer.

fluid dynamics effects play a huge role in how things move and interact with fluids like air and water. Drag forces try to slow things down, lift forces can lift heavy objects off the ground, and turbulence mixes things up in chaotic but important ways. Understanding these effects helps scientists, engineers, and even sports enthusiasts design better vehicles, efficient energy systems, and more.

## II. PROBLEM DESCRIPTION

Schematic drawing of the two-dimensional physical model considered in the present study is shown in Fig. 1. The system consists of a square cavity with sides of length  $L$ , within which a semi-circular cylinder with a diameter of  $0.4L$  is located at the centre of the cavity. The inner cylinder is kept at a constant electric potential  $\Phi_0 (> 0)$  and high temperature  $\theta$  while the outer cylinder is grounded, hence  $\theta_1=0$ . And outside the cavity with a low temperature  $\theta_1$ .

When an electrode (semi-circle) is placed in a liquid, having a square enclosure, which is having high potential at its centre. Charged particles are created the interaction between the solid surface and liquid charged particles, and then spread into the liquid. The charges are responsible for the variation in the flow patterns and temperature distribution and charge distribution.

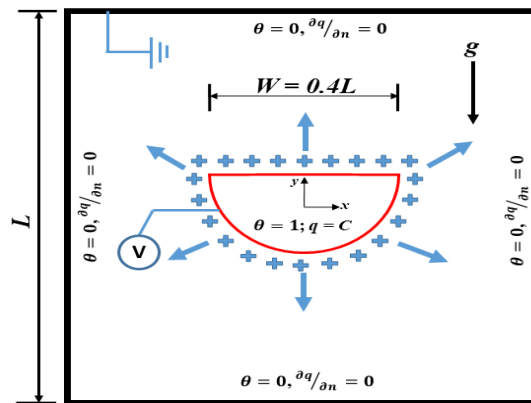


Figure 1: square enclosure inside a semi-circle

### 2.2 Lattice Boltzmann method for Electro thermo hydro- dynamics (ETHD):

Lattice Boltzmann Method (LBM) is a mathematical approach used to simulate and study fluid flow and other physical phenomena at a microscopic level. It's based on solving mesoscopic equation. LBEs are developed in a generalized lattice Boltzmann framework for the flow field, temperature variation charge distribution respectively.

$$c_j = \begin{cases} (0,0) & j = 0 \\ c \left( \frac{\cos(j-1)\pi}{2}, \sin \left[ \frac{(j-1)\pi}{2} \right] \right) & j = 1 - 4 \\ \sqrt{2}c (\cos[2j-1]\pi/4), \sin[ (2j-1)\pi/4] & j = 5 - 8 \end{cases}$$

**Streaming Speed (c):** The streaming speed is defined as how far particles move in one time step on the lattice grid. It's calculated as the ratio of the lattice size ( $\Delta x$ ) to the lattice time step ( $\Delta t$ ).  $\Delta x$  and  $\Delta t$  being the lattice size and the lattice time step.

$$C=\Delta x/\Delta t$$

The weight function  $\omega_j (X,j)$  is associated with each of the nine velocity directions (j) in the D2Q9 method . This function determines weight of particles moving in a specific direction.

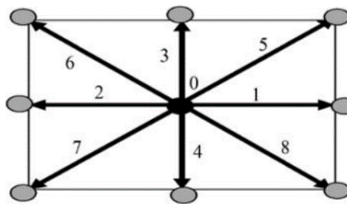
$$\omega_j = \frac{4}{9} \text{ for } j=0, \quad \omega_j = \frac{1}{9} \text{ for } j=1 - 4, \quad \text{for } \omega_j = \frac{1}{36} \text{ for } j=5 - 8$$

A set of mathematical equations (LBEs) within the lattice Boltzmann method to simulate various physical properties, including fluid flow, electric fields, charge distribution, and temperature. We use the SRT (single relaxation time) model and equilibrium distributions as references to achieve accurate simulations. The macroscopic quantities are then calculated from these distribution functions to represent the observable behaviour of the system.

The distribution function is the important parameter to characterize the effect of the molecules; what percentage of the molecules in a certain location of a container have velocities within a certain range, at a given instant of time.

### D2Q9

This model is very common, especially for solving fluid flow problems. It has high velocity vectors, with the central particle speed being zero .The speeds are  $c(0,0)$  ,  $c(1,0)$  ,  $c(1,1)$  ,  $c(0,1)$  ,  $c(-1,1)$  ,  $c(-1,0)$  ,  $c(-1,-1)$  ,  $c(0,-1)$  ,  $c(1,-1)$  for  $f_0; f_1; f_2; f_3; f_4; f_5; f_6; f_7$  and  $f_8$ ; respectively. The weighting factors for corresponding distribution functions are  $4/9, 1/9, 1/9, 1/9, 1/9, 1/36, 1/36, 1/36,$  and  $1/36$  .



**Fig 2:** Lattice arrangements for 2-D problems, D2Q9.

$$f_j (X+ c_j\Delta t, t+\Delta t) - f_j(x,t) = -\frac{1}{\tau_v} [f_j(x,t) - f_j^{eq} (x,t)] + \Delta t * f_j$$

$$g_j (X+ c_j\Delta t, t+\Delta t) - g_j(x,t) = -\frac{1}{\tau_\phi} [g_j(x,t) - g_j^{eq} (x,t)] + \Delta t * F_s$$

$$h_j (X+ c_j\Delta t, t+\Delta t) - h_j(x,t) = -\frac{1}{\tau_q} [h_j(x,t) - h_j^{eq} (x,t)]$$

$$l_j (X+ c_j\Delta t, t+\Delta t) - l_j(x,t) = -\frac{1}{\tau_\theta} [l_j(x,t) - l_j^{eq} (x,t)]$$

where  $f_j, g_j, h_j$  and  $l_j$  are the distribution functions for the flow field, electric potential, charge density distribution and temperature, respectively.

$$\tau_v = 3v/c^2\Delta t + 1/2$$

$$\tau_\phi = 3\gamma/c^2\Delta t + 1/2$$

$$\tau_q = 3D/c^2\Delta t + 1/2$$

$$\tau_\theta = 3\chi/c^2\Delta t + 1/2$$

These are relaxation time for all lattice boltzmann equations

$$f_j^{eq} = \rho \omega_j \left( 1 + \frac{c_j \cdot u}{c_s^2} + \frac{(c_j \cdot u)^2}{2c_s^4} - \frac{u^2}{2c_s^2} \right)$$

$$g_j^{eq}(x,t) = \omega_j \phi$$

$$h_j^{eq}(x,t) = q \omega_j \left\{ 1 + \frac{c_j (KE+u)}{c_s^2} + \frac{[c_j (KE+u)]^2 - c_s^2 (KE+u)^2}{2c_s^4} \right\}$$

$$l_j^{eq} = \theta \omega_j \left[ 1 + \frac{c_j \cdot u}{c_s^2} + \frac{(c_j \cdot u)^2}{2c_s^4} - \frac{u^2}{2c_s^2} \right]$$

$$F_j = \omega_j \left( 1 - \frac{1}{2\tau_v} \right) \frac{c_j [qE + \rho g \beta (\theta - \theta_{ref})]}{c_s^2}$$

$$S_j = \frac{\omega_j \gamma q}{\epsilon}$$

$$\rho = \sum_j f_j, \rho u = \sum_j c_j f_j + \frac{\Delta t}{2} (qE + \rho g), \varphi\theta = \sum_j g_j,$$

$$E = \frac{1}{\tau_\varphi c_s^2 \Delta t} \sum_j c_j g_j, q = \sum_j h_j, \theta = \sum_j l_j$$

### III. RESULTS AND DISCUSSION

T=100

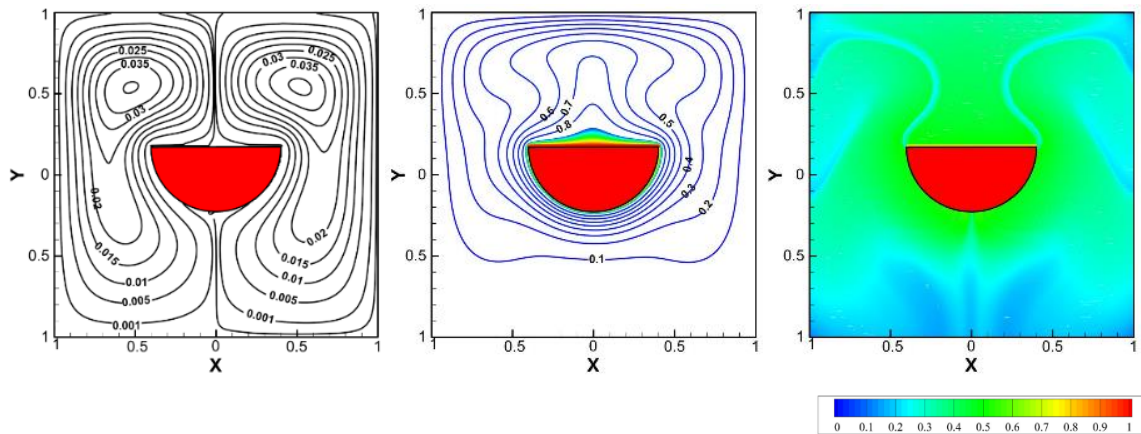


Fig. 3: It represents (left side) stream lines, (middle) temperature distribution, charge distribution(right).

T=300

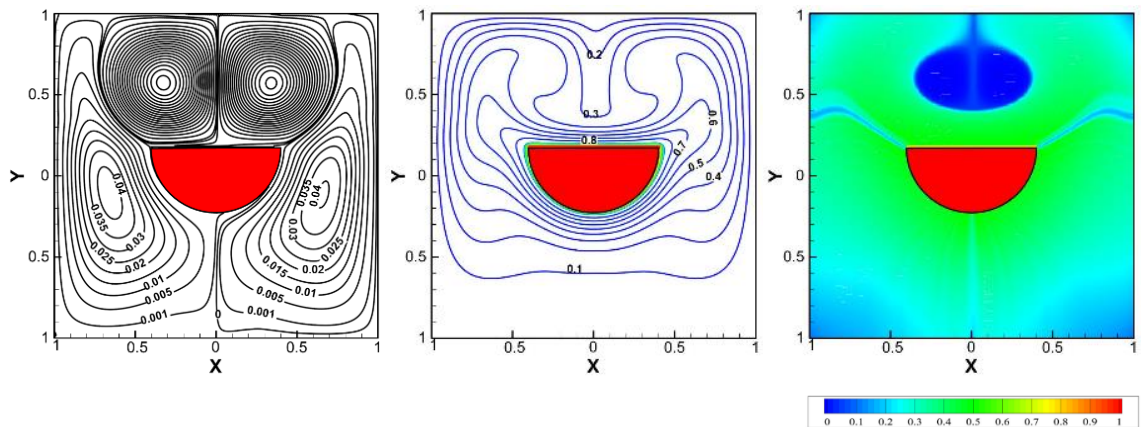


Fig. 4: It represents (left side) stream lines, (middle) temperature distribution, charge distribution(right).

T=500

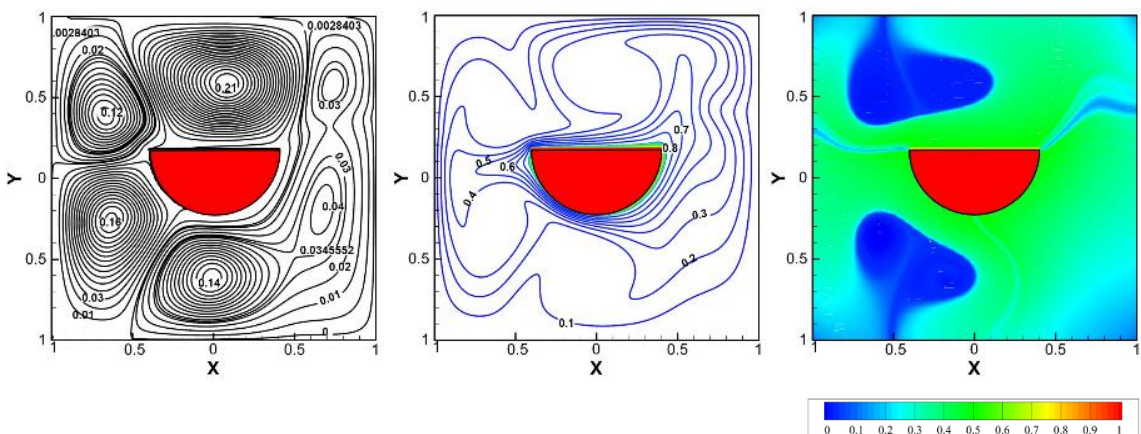


Fig. 5: It represents (left side) stream lines, (middle) temperature distribution, charge distribution(right).

### 3.1 Investigation:

The objective of this study is to investigate the impact of an external electric field on heat transfer enhancement. This effect arises from the interaction between natural convection heat transfer and electroconvection induced by injection. In all numerical simulations, the parameters related to the physical properties of liquid and injection strength are fixed at number (Pr) = 10, Mobility (M) = 10, Injection strength (C)= 10 and results are presented for different values of the electric Rayleigh number (T). In these simulations, a grid resolution of 300×300 is used for all cases.

To understand this, we imagined a scenario that there's a difference in temperature between an inner semi-circular cylinder that it is hotter, and an outer square enclosure that it is cooler. This kind of natural heat movement has been studied before.

### 3.2 Effect of electric Rayleigh number (T) on stream lines, temperature and charge distribution

In general, the heated lighter fluid is lifted and moves upward along the hot surface of the inner cylinder and the vertical symmetry line until it encounters the cold top wall. Then the fluid becomes gradually colder and denser while it moves horizontally outward in contact with the cold top wall. Consequently, the cooled denser fluid descends along the cold side walls. At T = 100, the imposed electric field is trivial and hence, the heat transfer is predominantly influenced by buoyancy force due to temperature difference. Consequently, two vortices which are termed as primary vortices form at right and left of the enclosure, and this is observed to be symmetric. As mentioned earlier, the cold fluid interacts initially with the curved edge of the semi-circular body, and therefore, the isotherms are seen to be denser at this location. At the flat edge, according to the flow pattern, the temperature lines are observed to be elongated at the middle of the heated body. Besides, the charge distribution plot indicates that charges distribute rapidly from the charged semi-circular cylinder to the enclosure walls, and at the corners of the cylinder, the transfer of charge particles are relatively faster than the other locations.

At T=300 the imposed electric field now plays a significant role in the heat transfer process. At this T, formation of secondary vortices is observed at top region of the enclosure as electric force is dominant. At higher T, the electric field proposes a stronger influence on the fluid behaviour. This interaction can cause the fluid to respond in a more complex manner, leading to the development of secondary vortices.

The secondary vortices intensity is elevated with T, and consequently, the kinetic energy of the primary vorticity intensity is reduced. This is evident from the reduced values of stream function values of primary vortices. The secondary vortices transport the heat from flat surface of the semi-circular body. Owing to this phenomenon, the isotherm distribution experiences a dip at centre portion of the semi-circular body. In fact, the charge distribution which has happened drastically at top region of enclosure indicates a bubble-like structure. This structure is the reason for such flow and thermal traits.

At T =500 In The streamlines more vortices are formed, This indicates that the fluid is flowing in a very complex pattern. The electric field is disrupting the fluid flow and creating vortices. These are responsible for the grater mixing of fluid, hence more transfer is observed. The vortices are more pronounced at an electric Rayleigh number of 500 than at an electric Rayleigh number of 300 because of the stronger electric field.

on increasing the value of T there is a formation of bubble is observed and the bubble is floating from right to left side initially it is started at the corners of the semi-circle and it is slowly is moving from the right corner to the left side of the wall. Due to this in temperature distribution on top and bottom corners of the left side wall is less whereas at middle portion high temperature gradient is observed. where with electric Rayleigh number 300, a small bubble is observed and intensity is small .and At T =500 one more bubble is formed. these are responsible for the more chaotic flow and more heat transfer.

### 3.3 Measure the Effect of electric Field on flow field $V_{max}/ Re_E$ :

In Fig 6  $V_{Max}$  represents maximum velocity  $V_{max} = \max(\sqrt{U_x^2 + U_y^2})$  and  $Re_E$  is electric Reynolds number, it is the ratio of electric Rayleigh number to the Mobility  $Re_E = T/M^2$ . The term  $V_{Max}/Re_E$  tells Measure the effect of electric field on flow field. Buoyancy force as well as applied electric force is responsible for the velocity. In the figure .6 with increment of electric field the IRJ is decreasing until it reaches electric rayleigh number

( $T=400$ ), as  $V_{MAX}/Re_E$  velocity value in the field reduces due to the suppression of the primary vortices due to secondary vortices.

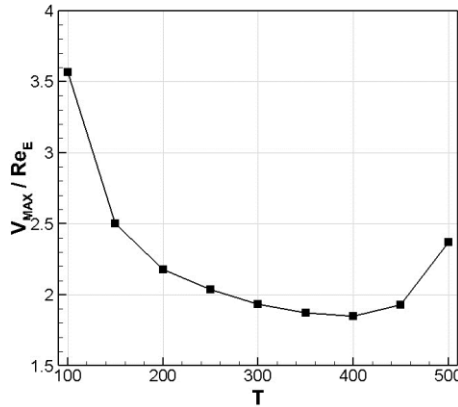


Fig. 6: Measure the effect of electric field on flow field

For higher the electric Rayleigh number ( $T=450$  and  $500$ ) the strength of electric field is increases when multiple vortices are formed due to multiple vortices formed therefore the  $V_{max}$  value is increases with reference to electric Rayleigh number ( $T$ ). Therefore there is a increasing in the trend in the graph. ( $T=450$  and  $500$ ).

at that time since all the vortices are having high velocity and maximum flow field at any region of the enclosure, so there is slight rising on the value of flow field from ( $T=400$ ). Upto  $450$  the fluid is experiences steady state, from ( $T=500$ ) the fluid is showing unsteady behaviour as strength of electric field is increases the flow motion is mainly driven by electric force, hence more convection heat transfer is observed.

**3.4 Effect of electric Rayleigh number on local Nusselt number value (Nu):**

In Fig.12 On the graph x- axis represents all the four sides of the square enclosure and y -axis represents the local Nusselt number In Fig.12 initially for electric Rayleigh number ( $T=100$ ) mostly the flow is driven by buoyancy force. the local Nusselt number value on the left side of the wall (DA) and right side wall (BC) having same value (3) as the primary vortices are symmetric around Y-axis hence heat transfer on both the walls are equal. whereas on the top wall (CD) for  $T=100$  the Nusselt number value increases and reaches a maximum of 8. In Fig.8 we have seen that in temperature distribution the isothermal lines are denser at top region. hence more temperature gradient is there at top side of the wall. which offers higher heat transfer rate At the bottom of the enclosure (AB) the local Nusselt number value is less due to gravitational force, hot or less denser fluid is moving upward direction and cold fluid is moving in the downward direction and hence temperature is low, therefore Nusselt number value is also less. on increasing the electric Rayleigh number ( $T = 300$ ) there is a rise in the value of Nusselt number on the side (DA) as there is a formation of secondary is observed, which increasing the kinetic energy of fluid. which influence the temperature traits in the enclosure, where convection heat transfer takes place consequently it influence the value of Nusselt number. there is a dip is observed on the graph for ( $T=300$ ) . On the top side (CD) due to formation of secondary vortices there is a temperature variation is observed due to this kinetic energy of the fluid is changes therefore the value of Nusselt number is decreasing As we increase the strength of the electric field, something interesting happens. we notice a sudden jump in the Nusselt value. This is because the fluid becomes more chaotic, with higher velocities inside the square enclosure. This leads to an increase in the fluid's kinetic energy, as more vortices form on the left side of the wall.

As a result, the Nusselt number, which measures heat transfer, goes up in the temperature distribution. Also, if we look at the isothermal lines (which indicate areas of equal temperature), they become denser at the top part of the flat surface shaped like a semi-circle. In this scenario, the fluid's motion is primarily driven by convection. The graph clearly shows that the highest Nusselt number value is reached at electric Rayleigh number 500. This means that, under these conditions, heat transfer is at its most efficient. This at a particular time step the graph is shown like this after some time at the bottom of the enclosure the Nusselt number value is increasing.

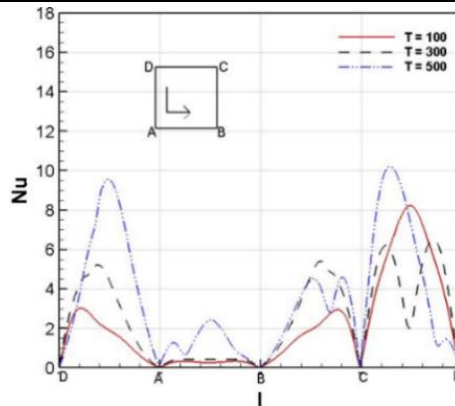


Fig .7: Variation of Nusselt number on four sides of square enclosure

3.5 Variation of Average Nusselt number on four sides of the enclosure:

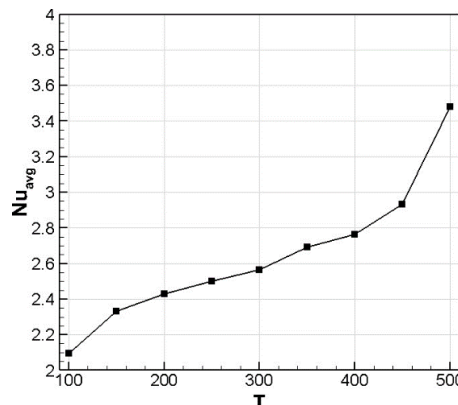


Fig .8: variation of average Nusselt number with respect of T.

In Fig .8 The graph shows that the variation of average Nusselt number with respect of T. from electric Rayleigh number 100 average Nusselt number value ( $Nu_{avg} = 2.095439$ ) is increasing at constant proportion, this indicates that fluid temperature is rising as kinetic energy of the fluid is increasing as fluid motion is driven by electric force ,once fluid is experiencing some electric force, the value of Nusselt number is slowly increasing ,from( $T=200$ ) there is a uniformly increasing in the value of Nusselt number in isothermal lines we have seen that temperature is uniformly distributed the secondary vortices is uniform around y-axis. Fluid is experiencing uniform motion. on increasing the value of electric Rayleigh number intensity of stream lines is increases and more vortices is formed ,the are responsible for the more temperature gradient and temperature traits, at  $T=300$  the Nusselt number value is(  $Nu = 2.564347$ ) compared to 100 there is a increasing in the value Nu on increasing electric Rayleigh number, the vortices which are termed as secondary are reduces the intensity of primary vortices.

IV. CONCLUSION

In the present work, the electro-thermo-hydro dynamics phenomenon in a square enclosure with an inner semi-circular cylinder is numerically investigated using a lattice Boltzmann method (LBM). the influence of an imposed electric field. At  $T=100$ , buoyancy forces primarily govern the heat transfer, As the electric Rayleigh number increases to  $T=300$ , the imposed electric field becomes a significant factor in the heat transfer process. This leads to the formation of secondary vortices heat transfer is driven by the dominance of the electric force. These secondary vortices play a crucial role in transporting heat from the flat surface of the semi-circular body. At  $T=500$ , the complexity of the fluid flow pattern increases further, with the formation of additional vortices. The disruptive effect of the electric field on the fluid flow leads to a chaotic flow pattern, resulting in enhanced heat transfer. for  $T=100$ , the local Nusselt number is lower due to the upward movement of hot fluid and the downward movement of cooler fluid is lesser , resulting in lower temperatures and subsequently, a lower Nusselt number. At  $T=300$ , the formation of secondary vortices is observed These vortices enhance the kinetic energy of the fluid, influencing temperature distribution and consequently, the convection heat transfer process,



leading to an increase in the Nusselt number. The graph clearly demonstrates that the highest Nusselt number value is achieved at an electric Rayleigh number of 500. This signifies that, under these specific conditions, the heat transfer process is at its most efficient. average Nusselt number is also varying indicates a more efficient convection heat transfer within the enclosure. These findings highlight the direct correlation between temperature and convection heat transfer, with increasing temperatures leading to improved heat transfer rates. . At higher ( $T=450$  and  $500$ ), where the strength of the electric field is increased, leads to a notable rise in the  $V_{Max}/ReE$  ratio. This is because all the vortices exhibit high intensity and maximum flow field at various regions within the enclosure. These findings emphasize the significant impact of the electric field strength on the flow field it is found that for sufficiently high values of  $T$ , the heat transfer rate is increases. Finally, the fully coupled electro-thermo-convection problem is investigated. It is found that the heat transfer enhancement due to the electric field and charge injection becomes significant only when the electric driving parameter  $T$  is varying from ( $T=450$  and  $500$ ).

The present study demonstrates the capability and high accuracy of the LBM in simulating the electro-convective and electro-thermo-convective flows with semi-circle shape in square enclosure. In a future work, we plan to extend our studies in understanding the unsteady flow patterns for further increasing the electric Rayleigh number and for other physical model.

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