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A NOVEL APPROACH TOWARDS GRAVITATIONAL AND BOUNDARY CONFORMAL FIELD THEORIES

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ABSTRACT

The Einstein field equations may be solved with a negative cosmic constant (Λ) using anti-de-Sitter spacetime. It has various unique properties, including a persistent negative curvature. Like Minkowski spacetime, it is maximally symmetric. Although Minkowski spacetime is flat, AdS spacetime has negative curvature. AdS spacetime has unique global features that are critical for the AdS/CFT correspondence. Asymptotically AdS spacetime approaches AdS geometry at spatial infinity (boundary). Many call this boundary the conformal boundary. The Boundary Conformal Symmetry The AdS spacetime border is conformal. This symmetry is essential for the AdS/CFT correspondence, which links the bulk gravitational theory (AdS) to the boundary conformal field theory (CFT). This duality is useful in theoretical physics, especially in tightly coupled quantum field theories. AdS spacetime and covering space are distinct. Often called the universal cover, AdS covers. The universal cover "covers" AdS spacetime with a simple linked space. Mathematically helpful but not as physically significant as AdS spacetime. The AdS spacetime is crucial to the AdS/CFT connection. knowing the holographic duality between the bulk gravity theory in AdS and the border CFT requires knowing its global characteristics, particularly its asymptotic behavior and conformal symmetry at the boundary. AdS spacetime, a negatively curved spacetime with unique global features, is crucial for the AdS/CFT correspondence. Its conformal boundary, conformal symmetry, and asymptotic behavior at infinity enable the duality between gravitational theories in the bulk and boundary conformal field theories, answering some of the most fundamental questions in theoretical physics, such as quantum gravity and strongly coupled field theories.

Keywords: Ads Spacetime, Conformal Field Theory, Minkowski Spacetime, Quantum Gravity.

I. INTRODUCTION

Anti-de Sitter spacetime (AdS) is a concept in the field of theoretical physics and general relativity. It is a specific solution to Einstein's field equations, and it plays a crucial role in various areas of theoretical physics, including string theory, AdS/CFT correspondence, and the study of black holes. Mathematical Description: AdS spacetime is a maximally symmetric Lorentzian manifold characterized by a constant negative curvature. Mathematically, n-dimensional AdS spacetime, denoted as AdS_n, can be described by the following metric:

$$ds^2 = \frac{1}{\cos^2(\rho)}\left(-dt^2 + d\rho^2 + \sin^2(\rho)d\Omega_{n-2}^2\right)$$

Here, (t) is time, (ρ) is a radial coordinate, and ($d\Omega_{n-2}^2$) represents the metric of the unit (n-2)-sphere. The constant negative curvature is evident in the term with ($\cos^2(\rho)$).

Negative Curvature: The constant negative curvature of AdS spacetime is what distinguishes it from flat spacetime (Minkowski space) and positive curvature spacetime (de Sitter space). This negative curvature means that AdS spacetime has a "saddle" shape, similar to a hyperbolic surface.

Boundary at Infinity: AdS spacetime has a conformal boundary at spatial infinity, denoted as (∂AdS_n). This boundary is important in the context of the AdS/CFT correspondence fig. 1, where physics in AdS is related to conformal field theories (CFTs) on the boundary.

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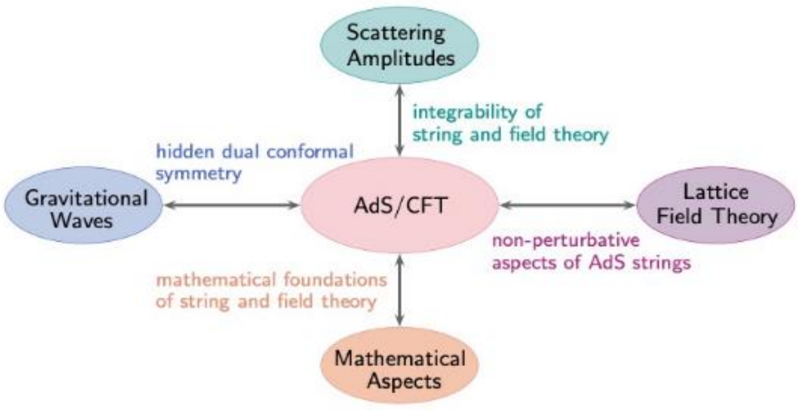

Figure. 1 AdS/CFT correspondance

Poincaré Coordinates: In some cases, it is more convenient to work with Poincaré coordinates for AdS spacetime. In these coordinates, the metric takes a different form that simplifies certain calculations. The Poincaré patch represents a global "slice" of AdS spacetime.

AdS/CFT Correspondence: One of the most remarkable aspects of AdS spacetime is its connection to conformal field theories through the AdS/CFT correspondence. This is a conjectured duality that relates a gravitational theory in AdS to a non-gravitational conformal field theory on the boundary. It has profound implications for our understanding of quantum gravity and the unification of fundamental forces.

Black Holes in AdS: AdS spacetime can contain black holes. These black holes have a negative cosmological constant and behave differently from their asymptotically flat counterparts. Black holes in AdS have a Hawking-Page phase transition, where they can transition between a thermal gas of particles and a black hole phase.

Cosmological Implications: While AdS spacetime itself is not a good model for our observed universe (which has a positive cosmological constant leading to expansion), the study of AdS can provide insights into aspects of cosmology and the nature of spacetime.

Higher Dimensions: AdS spacetime can exist in any number of dimensions, denoted as AdS_n. Higher-dimensional AdS spaces are relevant in string theory and related fields, where extra dimensions beyond our familiar three spatial dimensions are considered.

Negative Energy Density: AdS spacetime has a negative energy density associated with the negative cosmological constant. This negative energy density can be thought of as the "cost" of maintaining the constant negative curvature.

AdS/CFT (Anti-de Sitter/Conformal Field Theory) correspondence, "p space" typically refers to momentum space in the conformal field theory (CFT) on the boundary. This is analogous to "x space," which refers to the position or coordinate space in the CFT. In AdS/CFT, "x space" or position space refers to the space of boundary coordinates in the conformal field theory. It describes the spatial coordinates of the CFT, where fields are defined, and physical observables are studied. The CFT lives on the boundary of the Anti-de Sitter (AdS) space. "p space" refers to momentum space in the CFT. Just as you can represent the CFT's fields and observables in terms of positions in "x space," you can also describe them in terms of momenta or Fourier modes in "p space" as shown in Fig. 2. This is a mathematical representation that allows you to study the behavior of the CFT in terms of its momentum components rather than spatial coordinates. The relationship between "x space" and "p space" in AdS/CFT is similar to the relationship between position and momentum in quantum mechanics, where Fourier transforms are used to switch between representations. In AdS/CFT, understanding both position and momentum space is essential for exploring the dualities between the bulk AdS space and the boundary CFT. The choice of whether to work in "x space" or "p space" often depends on the specific problem you're trying to solve or the observables you're interested in. For certain calculations or analyses, it may be more convenient to work in one space or the other. The AdS/CFT correspondence allows physicists to gain insights into strongly coupled field theories (CFT) by mapping them to gravitational theories (AdS) in a higher-dimensional space. This duality has been instrumental in solving problems in quantum field theory, quantum

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gravity, and condensed matter physics, among other areas, by providing a new perspective on the relationships between different physical theories.

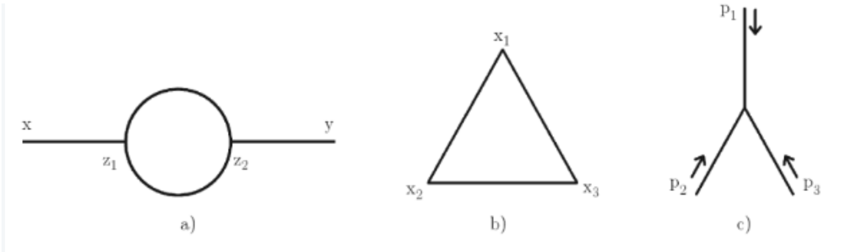

Figure. 2 1. A diagram of the setting sun in x space 2. A diagram of a triangle in x space 3. A diagram of a star in p space.

II. LITERATURE REVIEW

In the high N limit, some conformal field theories contain a sector representing supergravity, including Anti-deSitter spacetimes, spheres, and other compact manifolds. Our approach focuses on a subset of branes from M/string theory and studies the energy limit at which the brane's field theory becomes independent of the bulk. Large N demonstrates the reliability of near-horizon geometry. Supersymmetry generators in the superconformal group enhance near-horizon geometry's supersymmetry. Type IIB strings are included in the 4-dimensional N=4 super-Yang-Mills theory in the 't Hooft limit. According to our hypothesis, compactifications of M/string theory are dual to conformal field theories for several Anti-deSitter spacetimes. This results in a new definition of M-theory with five additional non-compact dimensions. The text highlights the importance of spacetime geometry in theoretical physics, especially in Albert Einstein's general relativity theory, where mass and energy affect the curvature and structure of spacetime as shown in fig. 3. We then summarize the string theory basis for the AdS/CFT correspondence and its extension to gauge/gravity duality. We emphasize the connection to quantum information theory by noting that the Fisher information metric of a Gaussian probability distribution is an Anti-deSitter space. We illustrate gauge/gravity duality with a holographic Kondo model, explaining its relevance to condensed matter physics. In this model, the interaction of a spin impurity with a free electron gas leads to low-energy screening, resulting in a logarithmic increase in resistivity. We investigate a large N variant of this model and develop its gravity dual counterpart, calculating power-law resistivity and entanglement entropy. We also explore quantum quenches and their connection to the Sachdev-Ye-Kitaev model. Next, we discuss an Einstein equations solution with a temporal singularity. We break down three-dimensional vectors and tensors into components along frame vectors. The development toward the singularity alternates between Kasner epochs, even in vacuum situations, due to spatial inhomogeneity.

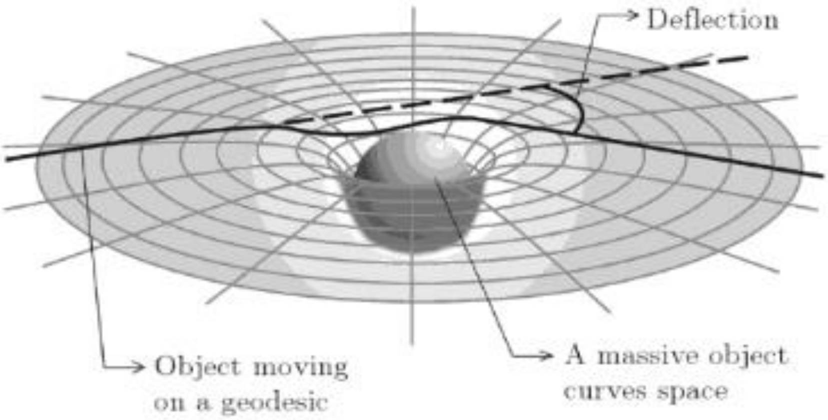

Figure. 3 A massive object curvatures space, and then matter (light objects) move on geodesics, being deflected and creating gravity.

These inhomogeneous terms mimic the function of matter energy-momentum in the Einstein equations. In another section, we summarize the current state of black hole thermodynamics, covering classical black hole thermodynamics, Hawking radiation, the extended second law, entropy boundaries, and methods for calculating black hole entropy. We then delve into a simplified generalization of classical thermodynamics for black holes. We explore Hawking's application of quantum theory, discussing thermal radiation, energetics of back-reaction, thermal Green functions, and their connection to entropy. Finally, we provide a high-level summary of the development of string theory and supersymmetry from the S-matrix theory program to superstring theory, a framework potentially unifying all forces, including gravity. The text acknowledges the absence of spacetime geometry details. These revisions aim to make the content more accessible and comprehensible to a broader audience while retaining the core concepts and ideas presented in the original text.

III. EXISTING SYSTEM

A system without n-dimensional Anti-de Sitter spacetime (AdS) is any physical or mathematical system that does not possess the geometric properties associated with AdS spacetime. Anti-de Sitter spacetime is a particular solution to Einstein's field equations in general relativity and is characterized by negative curvature. It has been extensively studied in the context of string theory and the AdS/CFT (Conformal Field Theory) correspondence. In the context of special relativity, flat spacetime, or Minkowski spacetime, is a system that does not have the negative curvature characteristic of AdS spacetime. It is a spacetime with zero curvature and is the basis for special relativity. The Schwarzschild spacetime is a spherically symmetric solution to Einstein's field equations that describes the spacetime outside a non-rotating, uncharged black hole. It is not AdS spacetime but rather a different solution. De Sitter spacetime is another solution to Einstein's field equations, but it is characterized by positive curvature. Unlike AdS, which has a negative cosmological constant, dS has a positive cosmological constant. It plays a role in cosmological models, such as the ΛCDM model, which describes the expansion of the universe. The gravitational fields around galaxies, stars, and other celestial bodies are described by general relativity, but these systems do not involve AdS spacetime. Instead, they have gravitational fields consistent with the mass and energy distributions within them. In quantum mechanics, physical systems are described by wave functions and operators, and they do not inherently involve spacetime geometries like AdS. Quantum mechanics and general relativity are two separate theories, and while there have been attempts to reconcile them (e.g., quantum gravity), quantum mechanical systems themselves do not inherently involve AdS spacetime. Classical mechanics, which describes the motion of macroscopic objects, also does not inherently involve spacetime geometries like AdS. Classical mechanics operates within the framework of Newton's laws of motion and is independent of general relativity. There are many physical and mathematical systems in which AdS spacetime does not play a role. The choice of spacetime geometry depends on the specific physical context and the gravitational effects being considered in a given system.

IV. PROPOSED SYSTEM

Relativists have continued to study n-dimensional Anti-de Sitter spacetime (AdS), one of the simplest curved spacetimes. Since the beginning of our field, it has given a test bed and easy instances for fresh ideas and classical and quantum spacetime notions. A striking aspect of the contemporary quest for a reformulation of theoretical physics, known as M- theory, is that many prior hypotheses fit in and are relevant to current efforts. This will be shown in detail. Its homogeneity and large isometry group make AdSn the perfect platform for studying the applicability of Wagnerian group-theoretic ideas in Minkowski spacetime En−1,1 with Poincare group E(n − 1, 1) to other spacetimes. Similar observations apply to energy-momentum and angular momentum conservation. Noether's theorem links the definition of the ADM mass in General Relativity and its positivity to the features of the isometry group.

In general, perturbation expansions require a background or "ground state." The ground state is perfectly symmetric in AdSn, de-Sitter spacetime, dSn with isometry group SO(n, 1), and Minkowski spacetime. In gauged supergravity theories, anti-de-Sitter spacetime is the natural ground state, whereas inflation studies naturally produce de-Sitter spacetime. As the cosmological constant approaches 0, flat space is a limit of de-Sitter spacetimes. An isometry group is contracted to a Poincaré group by a Wigner-Inonu contraction [2]. As a result of Lie algebra cohomology, these are the only isometry groups that can be generated this way [3].

Quantum gravity is interested in how global and topological characteristics of spacetime, such as closed timelike curves (CTCs) and spatial compactness, affect quantum theory. How geometrical and spacetime notions transfer into quantum mechanical terminology is a more fundamental concern. For de-Sitter and Anti-deSitter spacetimes, with space and time topologies Sn−1 R and Rn−1 S1, as well as strong symmetry, these problems may be put into group-theoretic terminology and have a straightforward solution. In this context, it is crucial to understand the numerous major distinctions between particle physicists' conventional compact Lie groups and Lorentzian spacetimes' non-compact isometry groups2.

Since Anti-de-Sitter space-times are the geometry of extreme black holes and extreme p-branes, essential to M-theory, they have received considerable attention. Maldacena's AdS/CFT correspondence hypothesis says that Euclidean Quantum gravity is what AdS is all about. A lot of attention has recently been paid to the mass and event horizon area of black holes that are not topologically simple. There is only one place where these black holes can form: Anti-de Sitter.

I will argue in the following comments that it is useful if not crucial to evaluate these contemporary challenges, like the previous ones, with the right global viewpoint, since doing so leads to startling and counter-intuitive findings. Due to the audience's curiosity, I'll focus on geometrical and group theoretic descriptions rather than supersymetry, supergravity, and supertstring theory. For an earlier report focusing on supergravity uses, see [5]. This gathering is ideal since much of the conversation can be expressed in basic mathematical terms that scientific professionals in this city, and potentially on this place, two and a half millennia ago would have understood.

M – Theory

No matter what it is, M-theory is a theory about p-branes, which are things with p spatial dimensions that move in high-dimensional spacetime, which is usually eleven dimensions. Point particles have p = 0, strings have p = 1, membranes have p = 2, and so on. An 'instanton' is the case where p = -1.

Levels of Description

We have different levels of description and approximations for branes in M-theory. Fundamental or F-string endpoints follow Dirichlet boundary conditions since they are D-branes. A completely quantum mechanical approach can be offered at this level through two-dimensional conformal field theory (CFT).

Soliton solutions of classical supergravity theories. This "heavy" brane approximation accounts for self-gravity and is applicable in the semi-classical approximation when several light branes, N, are stacked on top of each other. The initial solutions are usually static, have severe Killing horizons, and are BPS, allowing for supergravity theory's Killing spinor fields.

To answer the Dirac-Born-Infeld lagrangian in classical physics, a Σ(p+1)-dimensional submanifold λp+1 is used in a given spacetime setting M. Minimal submanifold equations make standard equations more flexible. Each D-brane has an embedding map x and an abelian gauge field A. The gauge field A gives scalar fields on the world volume Σ (p+1). This vector field is linked to the D-brane by a short, open string. A is a gauge field with no mass that is made when the string's length and energy almost disappear. Not all branes are on the above list because the M5-brain is not active.

Symmetry Enhancement

N branes equal N U(1) gauge fields. When branes converge, one may expect a U(1)N gauge theory over the world volume Σ¯ p+1. The string perspective reveals that N(N −1) additional "light states" are linked to virtually vanishing length threads starting on one of the N strings and finishing on another. This results in N2 massless gauge fields on Σ¯ p+1. A U(1)N2 gauge theory on Σ¯ p+1 may be expected to result from this. Non-abelian symmetry enhancement, previously only known via conformal field theory, is thought to occur, resulting in a non-abelian gauge group, U(N). The U(1) factor is linked to the D-brane's center of mass motion.

Killing spinors

The AdS/CFT correspondence, also known as the AdS/CFT duality or holographic duality, is a fundamental idea in theoretical physics that relates two seemingly different theories: Anti-de Sitter space (AdS) in gravity and conformal field theories (CFT) in quantum field theory. This duality has been a subject of extensive research in the fields of string theory, quantum gravity, and quantum field theory.

Killing spinors are related to the supersymmetry of a theory. In AdS/CFT, understanding the Killing spinors is important because it helps establish the connection between the supergravity theory in the bulk (the gravitational theory in the AdS space) and the boundary conformal field theory. Killing spinors are specific

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solutions to the supersymmetry transformations that preserve some of the supersymmetry of the theory. In AdS/CFT, the presence of Killing spinors in the bulk corresponds to certain conserved supercharges in the boundary CFT.

More specifically, the Killing spinor equations in AdS/CFT relate to the so-called BPS (short for Bogomolny-Prasad-Sommerfield) states. These are states in the boundary CFT that are related to the preservation of some fraction of the supersymmetry of the theory. The preservation of supersymmetry is a crucial aspect of AdS/CFT, as it helps establish the duality between the gravitational theory in AdS and the CFT on the boundary.

In practical terms, solving the Killing spinor equations in the AdS space can be quite challenging and depends on the particular AdS background and the specific supergravity theory under consideration. The exact form of these equations varies from one AdS/CFT correspondence to another, depending on the dimensionality of the AdS space, the type of supergravity theory, and the field content of the CFT.

To study Killing spinors in the context of AdS/CFT, one typically needs to start with the relevant supergravity action in the bulk and perform a detailed analysis of the supersymmetry transformations. The exact details can be quite complex and involve a lot of mathematical machinery. Researchers in the field of AdS/CFT often work on finding solutions to these equations to gain a deeper understanding of the duality and its implications for both gravitational and field theory physics. Killing spinors play an essential role in understanding the supersymmetry properties of the theories involved. Killing spinors are solutions to the Killing spinor equation, which is a set of differential equations that encode the supersymmetry transformations in a given spacetime. In the AdS/CFT correspondence, you typically have an Anti-de Sitter space on one side (gravity theory) and a conformal field theory on the other side (the boundary theory). Supersymmetry plays a crucial role in this correspondence because it relates the dynamics of these two theories.

Killing spinors in the context of AdS/CFT are solutions to the Killing spinor equation in the AdS spacetime. They are related to the preserved supersymmetries of the AdS background and are used to classify the different supercharges in the CFT. These supercharges are then associated with different operators in the boundary conformal field theory.

The precise details of Killing spinors in AdS/CFT can be quite involved, and they depend on the specific dimensions of AdS space, the amount of supersymmetry, and other parameters of the theory. The study of Killing spinors in the AdS/CFT context is an active area of research in theoretical physics, and it has led to significant insights into the relationship between gravity theories in AdS spaces and field theories on their boundaries.

Researchers use tools from supergravity and representation theory to study Killing spinors in AdS backgrounds. These investigations help uncover the symmetries and supersymmetry properties of the dual field theory and provide valuable insights into the AdS/CFT correspondence.

Supersymmetric solutions in supergravity theories include spinor fields ϵ fulfilling $\nabla\epsilon + N\epsilon = 0$ – (1), where ∇ is the Levi-Civita connection and N is a Clifford algebra valued one-form. The shape of N relies on the specifics of the supergravity hypothesis. Killing spinors must be covariantly constant if N = 0. The study of holonomy groups that stabilize spinors follows. The most well-known examples for relativists are pp-waves. Using AdSn, $N\alpha = \pm \frac{1}{2R} \Upsilon\alpha$ - (2), where α = 0, 1,..., n - 1. Either sign option yields the same number of solutions as in flat space. The conformally flat AdSn allows Killing spinors to meet the conformally invariant equation $\nabla\alpha\gamma\beta\epsilon + \nabla\beta\gamma\alpha\epsilon = \frac{1}{2n} g\alpha\beta\nabla^{\sigma} \gamma\sigma\epsilon$ - (3), which is the foundation of “Twistor theory”. Conformal Killing spinors naturally occur in conformal supergravity [16]. Recall that the theory of "Wave Geometry" was established at Hiroshima in the 1930s based on the assumption of solutions to an equation of the type (2).

V. CONCLUSION

Using anti-de-Sitter spacetime, it is possible to solve the Einstein field equations with a negative cosmic constant (). Among its many remarkable characteristics is a constant negative curvature. It is as perfectly symmetric as Minkowski spacetime. AdS spacetime is not flat like Minkowski spacetime but has a negative curvature instead. The unique global characteristics of AdS spacetime are essential to the AdS/CFT connection. AdS spacetime, asymptotically, converges on AdS geometry at the spatial horizon (border). The conformal boundary is a term used to describe such a limit. The Symmetry of the Boundary A conformal boundary characterizes AdS spacetime. The AdS/CFT correspondence, which connects bulk gravitational theory (AdS) with boundary conformal field theory (CFT), relies heavily on this symmetry. This duality has applications in theoretical physics, particularly in quantum field theories with strong coupling. Time in AdS and covering space are separate entities. The AdS cover is widely considered the standard blanket. Simply put, the universal cover "covers" AdS spacetime with a connected space. Quite useful mathematically, although it doesn't have nearly the physical weight of AdS spacetime. For the AdS/CFT correspondence to hold, the AdS spacetime is essential. Asymptotic behavior and conformal symmetry at the boundary are especially important for understanding the holographic duality between bulk gravity in AdS and the border CFT. The AdS/CFT correspondence relies on AdS spacetime, a negatively curved spacetime with distinctive global properties. Quantum gravity and strongly coupled field theories use its conformal boundary, conformal symmetry, and asymptotic behavior at infinity to provide answers to some of the most fundamental questions in theoretical physics.

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