e-ISSN: 2582-5208
International Research Journal of Modernization in Engineering Technology and Science
Volume:02/Issue:05/May-2020 www.irjmets.com

# QUANTUM LOGIC GATES: A DETAILED THEORETICAL STUDY OF THE QUANTUM NOT GATE

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## ABSTRACT

Quantum computers are meant to perform quantum computations. In order to understand how quantum computers perform quantum computations, we need to understand qubits' quantum states and how we can change those states. Quantum logic gates help do just that – change qubits' quantum states so that quantum computers can perform the desired quantum computations. This review paper is aimed at performing a detailed theoretical study of the Quantum NOT Gate.

**KEYWORDS**: Quantum Computing, Quantum Computer, Qubit, Quantum Bit, Quantum Logic Gate, Quantum NOT Gate.

## I. INTRODUCTION

Quantum logic gates serve as the basic building blocks for quantum computers to perform quantum computations. Quantum logic gates manipulate quantum information by changing the quantum state(s) of a qubit or a collection of them. Just as classical computers employ the use of classical logic gates such as the AND, OR, NOT gates and the likes in order to perform different types of computations, quantum computers use quantum logic gates in order to perform quantum information processing tasks and quantum computations.

## II. ANALYSIS

### a) Quantum NOT Gate: Algebraic Representation

Our first topic of discussion, the Quantum NOT Gate, is a generalization of the classical NOT Gate used in classical computers. For the computational basis states $|0\rangle$ and $|1\rangle$, the quantum NOT gate simply takes the state $|0\rangle$ as the input and produces the state $|1\rangle$ as the output; similarly, the state $|1\rangle$ as the input to the quantum NOT gate produces the state $|0\rangle$ as the output. The notation X is used as the general notation for representing a quantum NOT gate.

So, $X\,|0\rangle = |1\rangle$

$X\,|1\rangle = |0\rangle$

When the quantum NOT gate is applied to a general quantum superposition state $\alpha\,|0\rangle + \beta\,|1\rangle$ of a qubit, the quantum NOT gate acts linearly on the general quantum superposition state $\alpha\,|0\rangle + \beta\,|1\rangle$ and interchanges the $|0\rangle$ and $|1\rangle$ states.

$X\,(\alpha\,|0\rangle + \beta\,|1\rangle) = \alpha\,|1\rangle + \beta\,|0\rangle$

### b) Quantum NOT Gate: Quantum Circuit Representation

The Quantum Circuit representation of the X gate looks like the following.

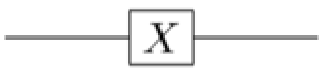


**Fig-1:** Quantum Circuit representation of X Gate.

It is also a common convention to explicitly specify the Input and Output quantum states in a Quantum Circuit, as depicted in the following figure.

$$\alpha|0\rangle + \beta|1\rangle \;—\boxed{X}—\; \alpha|1\rangle + \beta|0\rangle$$

**Fig-2:** Quantum Circuit representation of X Gate with the specified Input and Output quantum states.

**NOTE: -**

If you take a closer look at the Quantum Circuit representation depicted in Figure 2 above, the straight line that passes from left to right through the quantum NOT gate X is called a Quantum Wire and it represents a single qubit. The term 'Quantum Wire', the way it has been drawn in the quantum circuit representation and the way the overall quantum circuit itself is read or interpreted, it seems as if the qubit is moving from left to right through space. But that is not how the quantum circuit should be interpreted – instead, the quantum wire's left-to-right representation should be thought of as a representation of the passage of time. Thus, the segment of the quantum wire that sits to the left of the quantum NOT gate X should be strictly interpreted as representing only the passage of time and nothing is happening to the qubit as such. The quantum NOT gate X then gets applied to the qubit's input state. And, finally, the segment of the quantum wire that sits to the right of the quantum NOT gate X leads to the desired output state. In fact, the quantum circuit representation of the quantum NOT gate X clearly depicts that we have performed a quantum computation involving a single qubit and quantum logic gate.

**c) Quantum NOT Gate: Matrix Representation**

The following denotes a 2x2 matrix representation of a quantum NOT gate.

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

**Fig-3:** 2x2 Matrix Representation of the Quantum NOT Gate

Now, let us prove that the 2x2 matrix representation depicted in figure 3 above is indeed that of the quantum NOT gate. For that, let us take into consideration the general quantum superposition state $\alpha|0\rangle + \beta|1\rangle$, and note down the vector notations of the computational basis states **|0⟩** and **|1⟩**.

It is known that

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

**Fig-4:** Vector notations of the computational basis states **|0⟩** and **|1⟩**

Therefore,

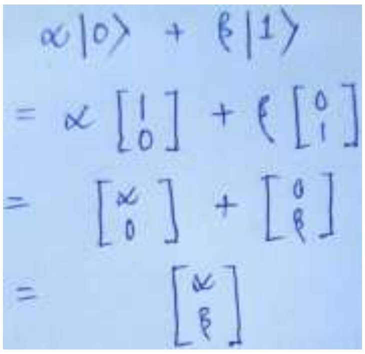

**Fig-5: G**eneral Quantum Superposition State α |0⟩ + β |1⟩ represented in a vector notation

So, we now have

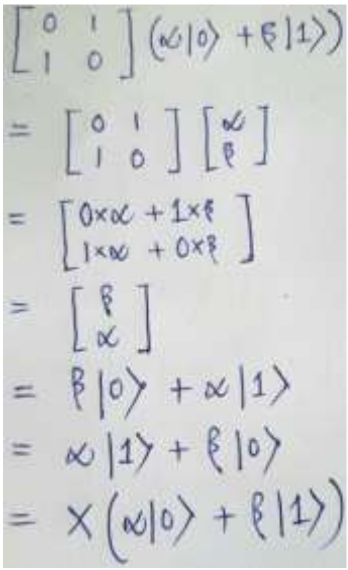

**Fig-6:** Mathematical proof of the 2x2 matrix representation of a quantum NOT gate

Therefore, it has been established that

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

**d) Simplest Quantum Circuit**

By now, you might be tempted enough to believe that the Quantum NOT gate X could be the simplest possible quantum circuit. But that is not the case. In fact, the simplest quantum circuit is a single quantum wire which does absolutely nothing. A single quantum wire quantum circuit is merely a representation of a single qubit being preserved in time. In other words, an arbitrary qubit quantum state |ψ⟩ (Greek alphabet, Psi), when input to a single quantum wire quantum circuit, outputs the same arbitrary qubit quantum state |ψ⟩.

**Fig-7:** The simplest Quantum Circuit – a single Quantum Wire

e-ISSN: 2582-5208
International Research Journal of Modernization in Engineering Technology and Science
Volume:02/Issue:05/May-2020 www.irjmets.com

**e) Multi X gate Quantum Circuit**

The following figure depicts a simple multi X gate quantum circuit that consists of two X gates in a row.


**Fig-8:** A multi X gate Quantum Circuit, comprising of two X gates in a row

Let us consider the general quantum superposition state α |0⟩ + β |1⟩ being input to the multi X gate quantum circuit depicted in figure 8 above.

X (X (α |0⟩ + β |1⟩))

= X (α |1⟩ + β |0⟩)

= α |0⟩ + β |1⟩

We can clearly see that a multi X gate quantum circuit, comprising of two X gates in a row, as depicted in figure 8 above, produces as output the same general quantum superposition state α |0⟩ + β |1⟩ that it had previously taken as its input. A multi X gate quantum circuit, comprising of two X gates in a row, as depicted in figure 8 above, thus, recovers the original quantum state and is therefore equivalent to a single quantum wire quantum circuit.


**Fig-9:** A multi X gate Quantum Circuit, comprising of two X gates in a row, as a Quantum Circuit is equivalent to a single quantum wire quantum circuit

If we input an arbitrary qubit quantum state |ψ⟩ to the multi X gate Quantum Circuit, comprising of two X gates in a row, as depicted in figure 8, the first X gate outputs the quantum state X |ψ⟩. The quantum state X |ψ⟩ is now the input to the second X gate, which outputs the quantum state XX |ψ⟩.

Now,

$$XX = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0\times0+1\times1 & 0\times1+1\times0 \\ 1\times0+0\times1 & 1\times1+0\times0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

**Fig-10:** Product of XX

Therefore, the product of XX is an Identity Matrix. By the property of the identity matrix operation, XX |ψ⟩ = |ψ⟩.

Let us assume that |ψ⟩ = a general quantum superposition state α |0⟩ + β |1⟩

Then,

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$
$$\Rightarrow |\psi\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$
$$XX|\psi\rangle = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$
$$= \begin{bmatrix} 1\times\alpha + 0\times\beta \\ 0\times\alpha + 1\times\beta \end{bmatrix}$$
$$= \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = |\psi\rangle$$

**Fig-11:** Mathematically proving that XX $|\psi\rangle = |\psi\rangle$

Thus, we have again proved that a multi X gate quantum circuit, comprising of two X gates in a row, as depicted in figure 8 above, is equivalent to a single quantum wire quantum circuit.

## III. CONCLUSION

This review paper provides the reader a detailed perspective of one of the simpler quantum logic gates – the quantum NOT gate X, which, we have seen is a generalization of the classical NOT gate used in the classical computers.

## ACKNOWLEDGEMENTS (optional)

The author sincerely bows down in all true respect, love and devotion before the ALMIGHTY GOD without whose grace and mercy, nothing is possible in this materialistic world.

The author wishes to express his true love for his better half, Mrs. Sukanya Basu Sarkar, who has always been a pillar of strength and inspiration to the author, and who probably is also one of the true and passionate critic and admirer of the author's work.

The author sincerely wishes his gratitude and love for the organization he is an extremely proud employee of, BMC Software India Pvt Ltd. I LOVE YOU, BMC!!!

And, finally, the author would like to express his humble and sincere gratitude to IBM Corporation, whose work on Quantum Computing and uncountable other technologies, has truly inspired the author to study and research on the field of Quantum Computing.

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