

ANALYSIS THE INTERNAL PRESSURE OF THE THIN CYLINDRICAL SHELL BY NEWTON'S FORWARD INTERPOLATION

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ABSTRACT

Whenever a cylindrical shell is subjected to an internal pressure, its walls are subjected to tensile stresses. This paper studies the internal pressure of the circumferential stress due to the subject of thin cylindrical shells. The research methodology of Newton's Forward Interpolation can be implemented to determine the exact internal pressure due to the function of exact circumferential stress or hoop stress of the thin cylindrical shells. The corresponding theoretical values of internal pressure were calculated using the findings in literature and compared with test results. Both the results were found matching with each other with a variation of just 0.32 approximately. Based on this validation lesson, the internal pressure was built-in the actual study.

Keywords: Thin Cylindrical Shell, Internal Pressure, Circumferential Stress, Newton's Forward Interpolation

I. INTRODUCTION

In engineering field, we daily move towards the vessels of cylindrical and spherical shapes containing fluids such as tanks, boilers, compressed air receivers, etc. Generally, the walls of such vessels are extremely thin as compared to their diameters. These vessels, when empty, are subjected to atmospheric pressure internally as well as externally. In such a case, the resultant pressure on the walls of the shell is zero. But whenever a vessel is subjected to internal pressure (due to steam, compressed air, etc.) its walls are subjected to tensile stresses.

In general, if the thickness of the wall of a shell is less than $\frac{1}{10}th$ to $\frac{1}{15}th$ of its diameter, it is known as a thin shell.

II. MALFUNCTION OF A THIN CYLINDRICAL SHELL DUE TO AN INTERNAL PRESSURE

Whenever a cylindrical shell is subjected to an internal pressure, its walls are subjected to tensile stresses. It will be interested to know that if these stresses exceed the permissible limit, the cylinder likely to fail in any one of the following two ways as shown below-

- a. It may split up into two troughs
- b. It may split up into two cylinders

III. STRESSES IN A THIN CYLINDRICAL SHELL

Whenever a cylindrical shell is subjected to an internal pressure, its walls are subjected to tensile stresses. The cylindrical shells subjected to tensile stresses by the following two ways-

- a. Circumferential Stress (or Hoop Stress)
- b. Longitudinal Stress

In case of thin shells, the stresses are assumed to be uniformly distributed throughout the wall thickness. But, in the case of thick shells, the stresses are no longer uniformly distributed and the problem becomes multifaceted.

IV. CIRCUMFERENTIAL STRESS (or HOOP STRESS)

Consider a thin cylindrical shell subjected to an internal pressure. We know that, as a result of the internal pressure, the cylinder has a tendency to split up into two troughs.

Consider,

L = length of the shell;

D = diameter of the shell;

T = thickness of the shell;

p = Intensity of internal pressure;

Total pressure along the diameter along X-X axis of the cylindrical shell,

P = Intensity of internal pressure \times Area

$P = p \times D \times L$;

Resisting Section = $2T \times L$ (of two sections);

Now, the circumferential stress in the shell,

$$\sigma_c = \frac{\text{Total Pressure}}{\text{Resisting Section}} = \frac{pDL}{2TL} = \frac{pD}{2T} \text{ (across X-X axis).....(i);}$$

If η is the efficiency of the riveted joints of the cylindrical shell, then the circumferential stress

$$\sigma_c = \frac{\text{Total Pressure}}{\text{Resisting Section}} = \frac{pDL}{2T\eta} = \frac{pD}{2T\eta} \text{ (across X-X axis).....(ii);}$$

Illustration 1: A cylindrical shell of 1.8 m diameter is made up of 15 mm thick plates. Find the internal pressure in the plates, if the boiler is subjected to a circumferential stress of 133 MPa. Take efficiency of the joints as 0.7

Computation 1: Here the data of the cylindrical shell are as follows-

Diameter of the shell (D) = 1.8 m = 1.8×10^3 mm;

Thickness of plates (T) = 15 mm;

Circumferential Stress (σ_c) = 133 MPa = 133 N/mm²;

Efficiency (η) = 0.7

We know that circumferential stress,

$$\sigma_c = \frac{\text{Total Pressure}}{\text{Resisting Section}} = \frac{pDL}{2TL} = \frac{pD}{2T} \text{ (across X-X axis)}$$

$$\sigma_c = \frac{\text{Total Pressure}}{\text{Resisting Section}} = \frac{pD}{2T\eta} \text{ (in case of efficiency)}$$

$$133 = \frac{p \times 1800}{2 \times 15 \times 0.7}$$

Therefore, internal pressure (p) = 1.55 MPa = 1.55 N/mm²

Table- 1: Following observation table of internal pressure (p) and circumferential stress (σ_c), when the parameters of diameter of **cylindrical shell** (D), **thickness** (T) and **efficiency** (η) are **constants** to the above illustration:

Number of Observations	Diameter of the shell (D)	Thickness of the shell (T)	Efficiency (η)	Circumferential Stress (σ_c)	Internal Pressure (p)
1.	1800 mm	15 mm	0.7	124 MPa	1.45 MPa
2.	1800 mm	15 mm	0.7	129 MPa	1.50 MPa
3.	1800 mm	15 mm	0.7	134 MPa	1.56 MPa
4.	1800 mm	15 mm	0.7	139 MPa	1.62 MPa
5.	1800 mm	15 mm	0.7	144 MPa	1.68 MPa
6.	1800 mm	15 mm	0.7	149 MPa	1.74 MPa
7.	1800 mm	15 mm	0.7	154 MPa	1.79 MPa
8.	1800 mm	15 mm	0.7	159 MPa	1.85 MPa
9.	1800 mm	15 mm	0.7	164 MPa	1.92 MPa
10.	1800 mm	15 mm	0.7	169 MPa	1.97 MPa

V. RESEARCH METHODOLOGY

Let $y = f(x)$ be a real valued function whose values are known for $(n + 1)$ equally spaced points, $x_0, x_1, x_2, \dots, x_n$ i.e.

$x_1 - x_0 = x_2 - x_1 = x_3 - x_2 = x_4 - x_3 = \dots = h$, then by Newton's Forward Interpolation Formula-

Let these values be shown in the following table:

x:	x₀	x₁	x₂	x_{n-1}	x_n
y:	y₀	y₁	y₂	y_{n-1}	y_n

Then for any x,

$$y = f(x) = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \dots + \frac{u(u-1)(u-2)\dots(u-n+1)}{n!}\Delta^n y_0$$

where $u = \frac{x-x_0}{h}$ and $\Delta y_0, \Delta^2 y_0, \Delta^3 y_0, \dots$ are the first order, second order, third order difference respectively.

5.1. Limitations:

- a. The formula is used only when the interpolating points are equally interval (i.e. h = equal interval)
- b. The formula is used to interpolate near the beginning side of the tabulated form.

Illustration 2: Following are the observation table of circumferential stress (σ_c) and internal pressure (p) with the help of data points:-

x = σ_c	124	129	134	139	144	149	154	159	164	169
y = p	1.45	1.50	1.56	1.62	1.68	1.74	1.79	1.85	1.92	1.97

Locate the internal pressure, when the circumferential stress (σ_c) is 133 MPa with the help of Newton's Forward Difference Interpolation method?

Computation 2: Let $y = p$ (internal pressure) = $f(x)$ be a function of circumferential stress having the following values:-

x = σ_c	124	129	134	139	144	149	154	159	164	169
y = p	1.45	1.50	1.56	1.62	1.68	1.74	1.79	1.85	1.92	1.97

Since 133 lies between 129 and 134, so we take here $x_0 = 129, x_1 = 134, x_2 = 139, \dots$ so on and $y_0 = 1.50, y_1 = 1.56, y_2 = 1.62, \dots$ so on

Note that $134 - 129 = 139 - 134 = \dots = 5$, that means the value of x are equally spaced with $h = 5$.

We are going to find the approximate internal pressure of $f(133)$, so here $x = 133$.

Here $u = \frac{133-129}{5} = 0.8$.

Now, we construct the differential table which is as follows:-

x = σ_c	y = p	Δp	$\Delta^2 p$	$\Delta^3 p$	$\Delta^4 p$	$\Delta^5 p$	$\Delta^6 p$	$\Delta^7 p$	$\Delta^8 p$	$\Delta^9 p$
124	1.45	0.05	0.01	-0.01	0.01	-0.01	0.00	0.05	-0.19	0.46
129	1.50	0.06	0.00	0.00	0.00	-0.01	0.05	-0.14	0.27	
134	1.56	0.06	0.00	0.00	-0.01	0.04	-0.09	0.13		
139	1.62	0.06	0.00	-0.01	0.03	-0.05	0.04			
144	1.68	0.06	-0.01	0.02	-0.02	-0.01				
149	1.74	0.05	0.01	0.00	-0.03					
154	1.79	0.06	0.01	-0.03						
159	1.85	0.07	-0.02							
164	1.92	0.05								
169	1.97									

According to Newton's Forward Difference Interpolation method-

$$y = f(x) = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \dots + \frac{u(u-1)(u-2)\dots(u-n+1)}{n!}\Delta^n y_0$$

$$= 1.50 + 0.8 \times 0.06 + \frac{0.8(0.8-1)}{2!} \times 0.00 + \frac{0.8(0.8-1)(0.8-2)}{3!} \times 0.00 + \frac{0.8(0.8-1)(0.8-2)(0.8-3)}{4!} \times 0.00 + \frac{0.8(0.8-1)(0.8-2)(0.8-3)(0.8-4)}{5!} \times (-0.01) + \frac{0.8(0.8-1)(0.8-2)(0.8-3)(0.8-4)(0.8-5)}{6!} \times (0.05) + \dots$$

$$\frac{0.8(0.8-1)(0.8-2)(0.8-3)(0.8-4)(0.8-5)(0.8-6)}{7!} \times (-0.14) + \frac{0.8(0.8-1)(0.8-2)(0.8-3)(0.8-4)(0.8-5)(0.8-6)(0.8-7)}{8!} \times (0.27) + \frac{0.8(0.8-1)(0.8-2)(0.8-3)(0.8-4)(0.8-5)(0.8-6)(0.8-7)(0.8-8)}{9!} \times (0.46)$$

$$= 1.546170902$$

$$= 1.55 \text{ (approx)}$$

VI. CONCLUSION

This paper deals with the internal pressure of the thin cylindrical shells due to the circumferential stress. Numerous internal pressure have been created and utilized over the thin cylindrical shells or vessels as per the experimental point of view. Based on the study, the observed advantages and disadvantages, as well as areas of application for various industries, Newton's Forward Interpolation along with the other numerical methods in their original forms, can be extremely successful in their applications to find the accurate result, but only if their strengths and weaknesses are properly assessed.

VII. REFERENCES

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